Directed Reachability for Infinite-State Systems

Michael Blondin[1], Christoph Haase[2], Philip Offtermatt[1,3]

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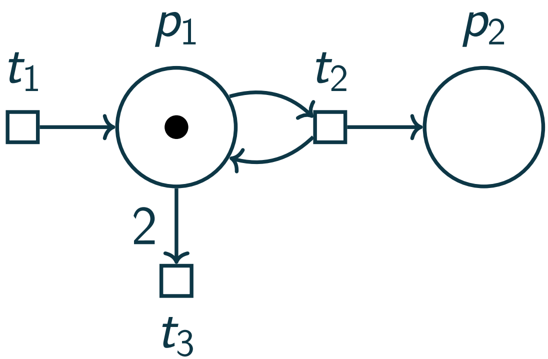

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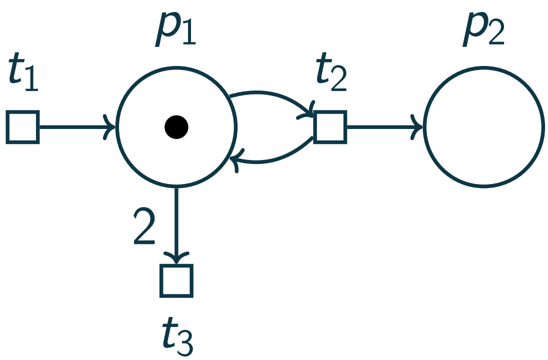

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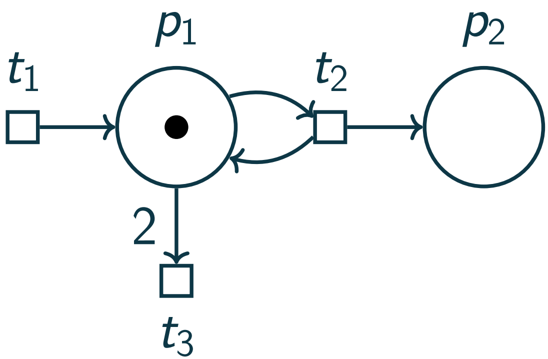
Marking: $P \to \mathbb{N}$

$(1, 0)$

p_1, p_2



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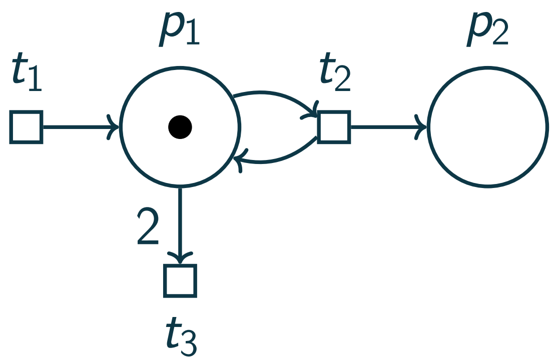

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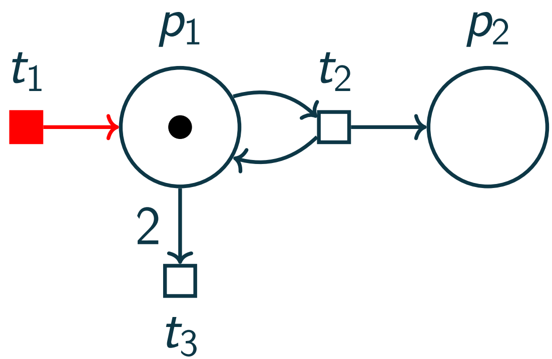

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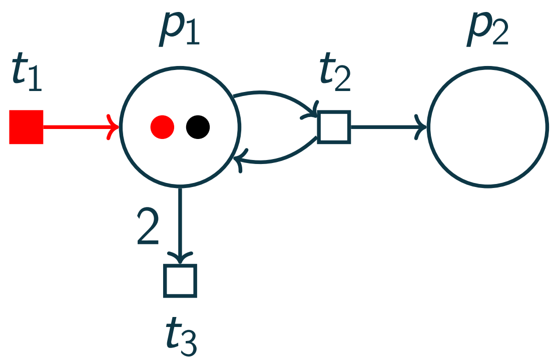

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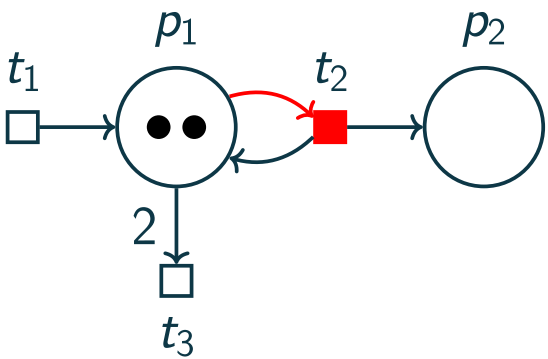

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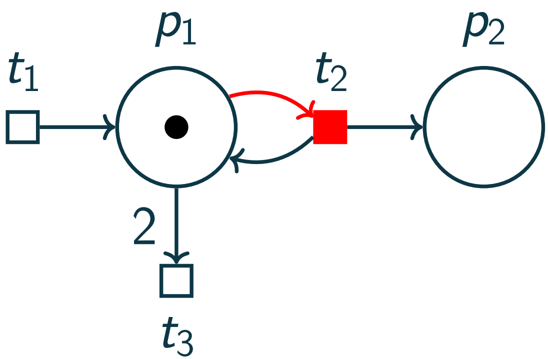

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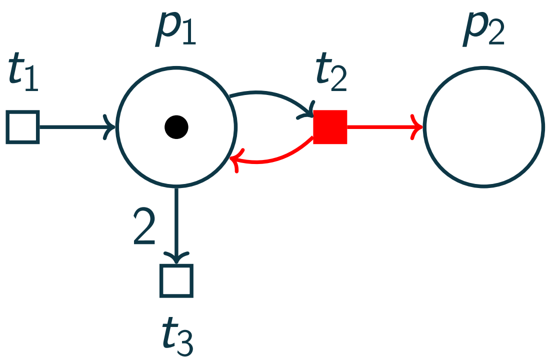

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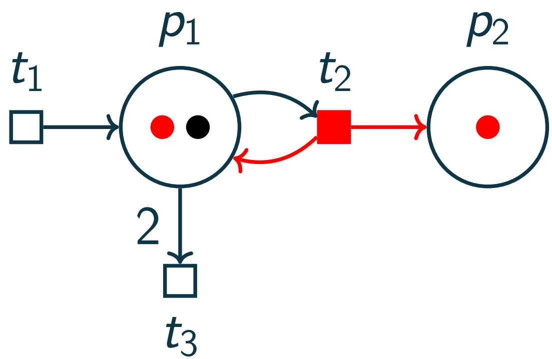

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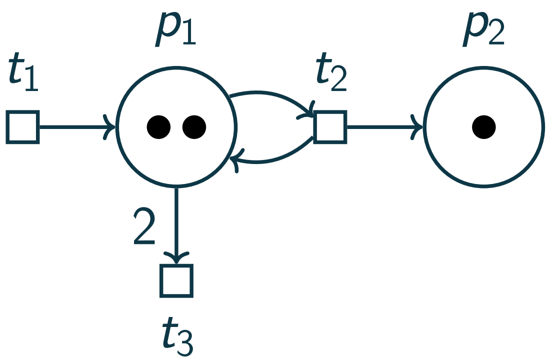

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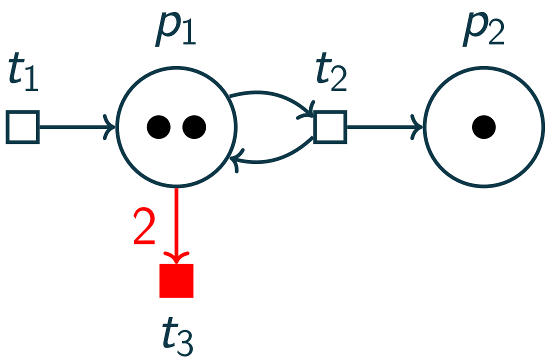

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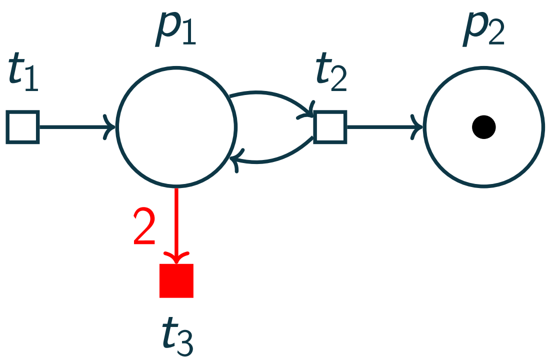

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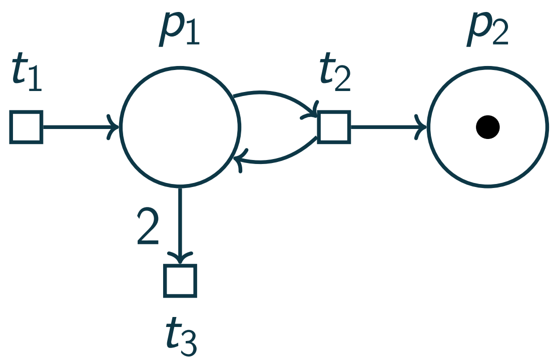

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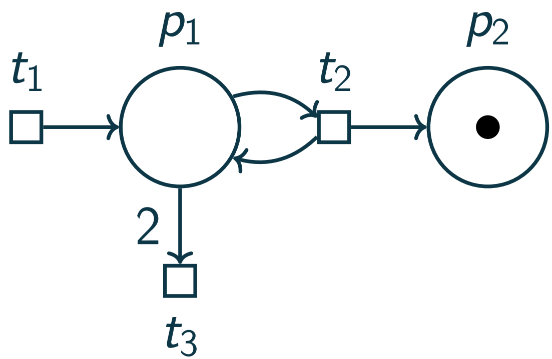

Petri nets finitely represent infinite-state systems:
Reachability Graphs

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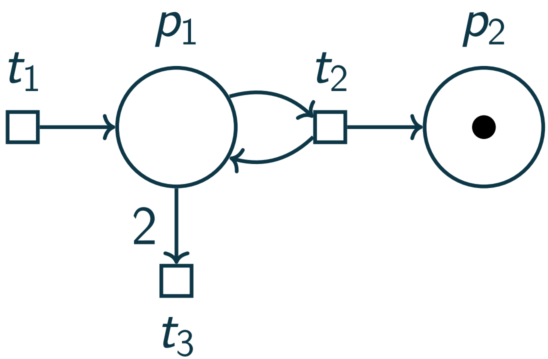

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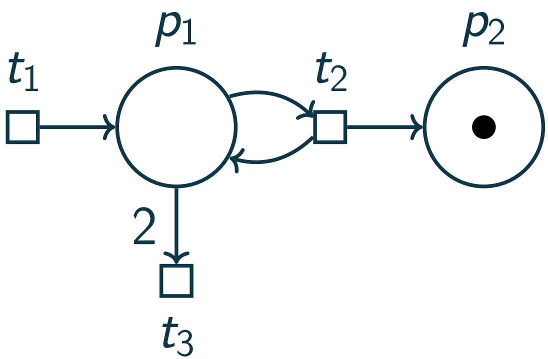

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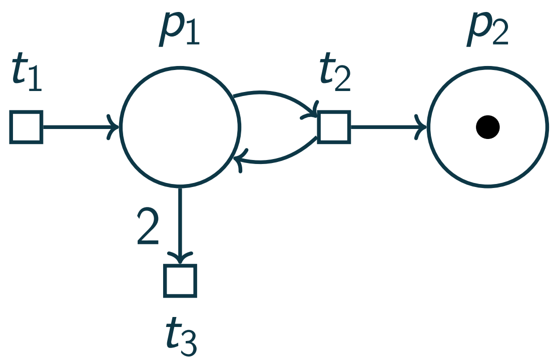

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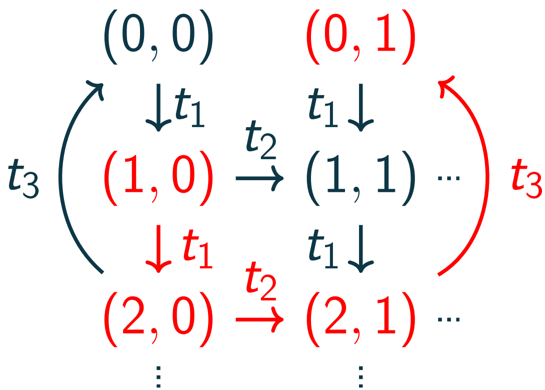

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Witnessing run:

$(1,0) \xrightarrow{t_1} (2,0) \xrightarrow{t_2} (2,1) \xrightarrow{t_3} (0,1)$

Problems for Petri Nets

Reachability: Is there a run that starts with m_{init} and ends with m_{target}? (e.g., $m_{init} = (1, 0), m_{target} = (0, 1)$)

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Non-elementary lower bound [Czerwiński et al., 2019]

Ackermannian upper bound [Leroux and Schmitz, 2019]

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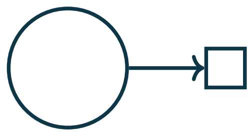
Reduction to reachability: Add transitions that delete tokens



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No practically efficient semi-procedures for showing reachability in infinite-state systems

Outline

- Part I: **Applications** - Why is this useful?
- Part II: **Approximations** - Relaxing Reachability
- Part III: **Directed Search** - Searching with Guidance
- Part IV: **Experiments** - Prototype Evaluation

Part I

Applications

Some Applications of Petri Nets

Concurrent Program Analysis

Many threads on same program

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`s`: Shared Boolean variable

Initially, `s`$=0$

```
def fun():
  s = 1
  s = 0
  if s == 1:
    raise Err()
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Can any thread reach the error state?

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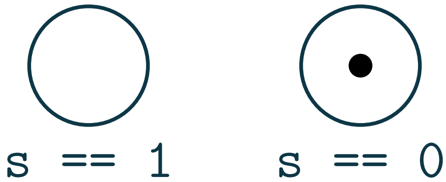

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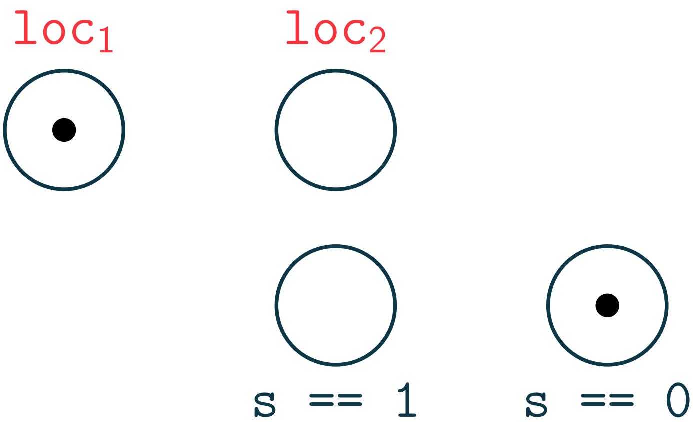

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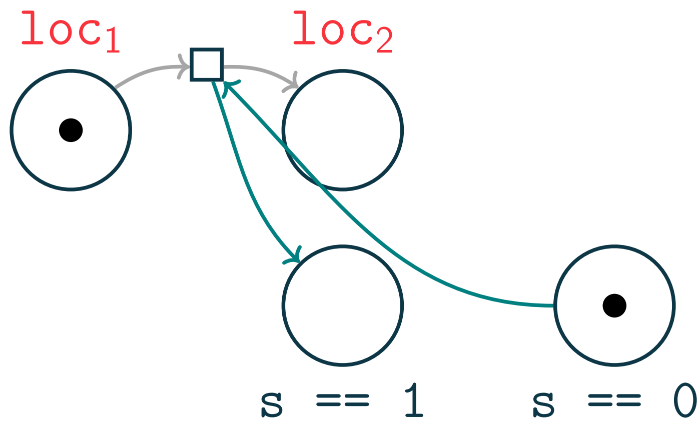

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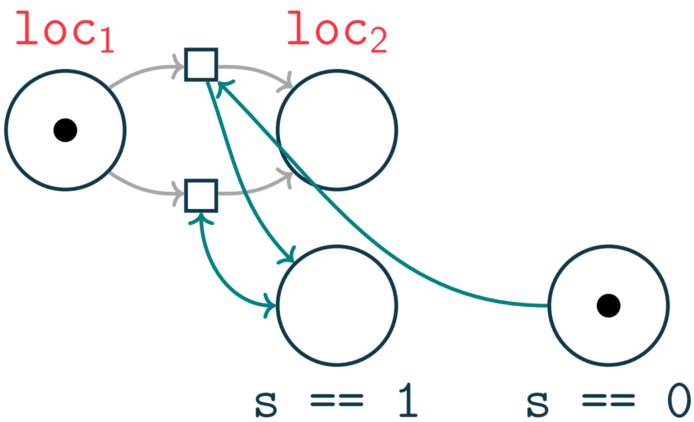

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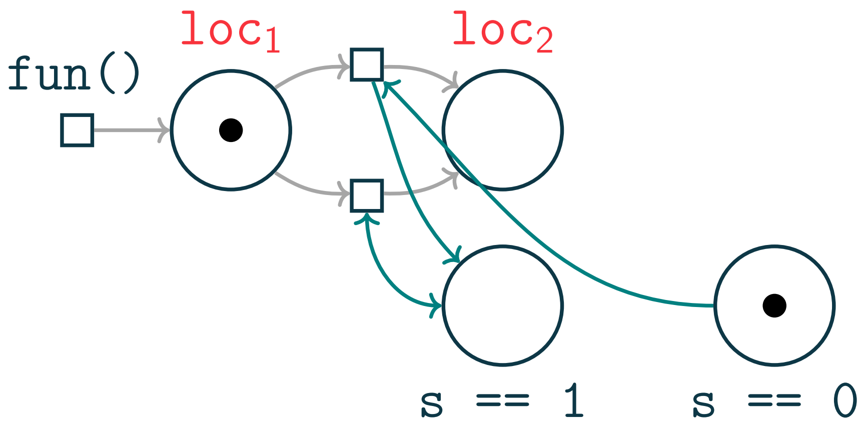

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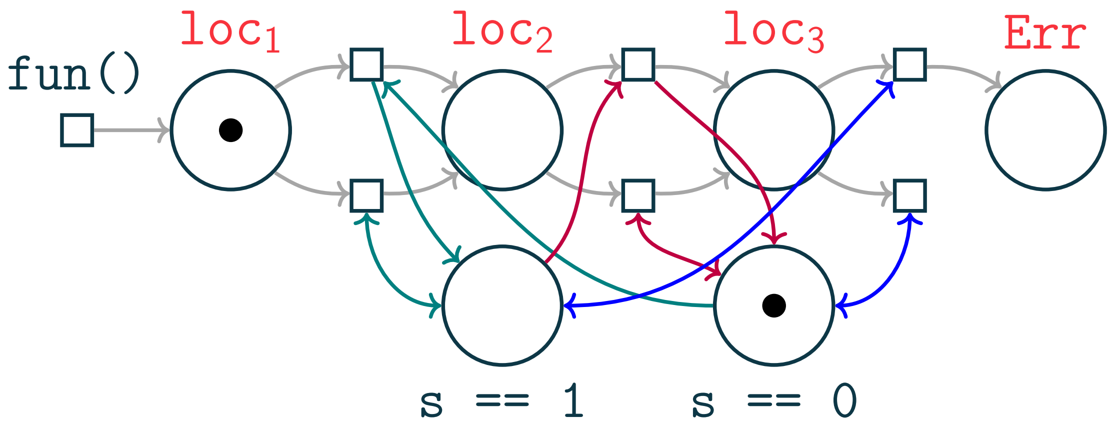

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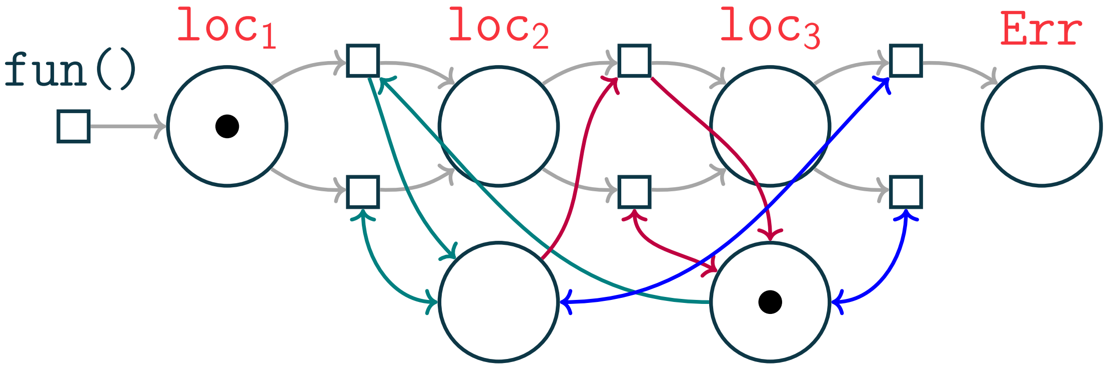

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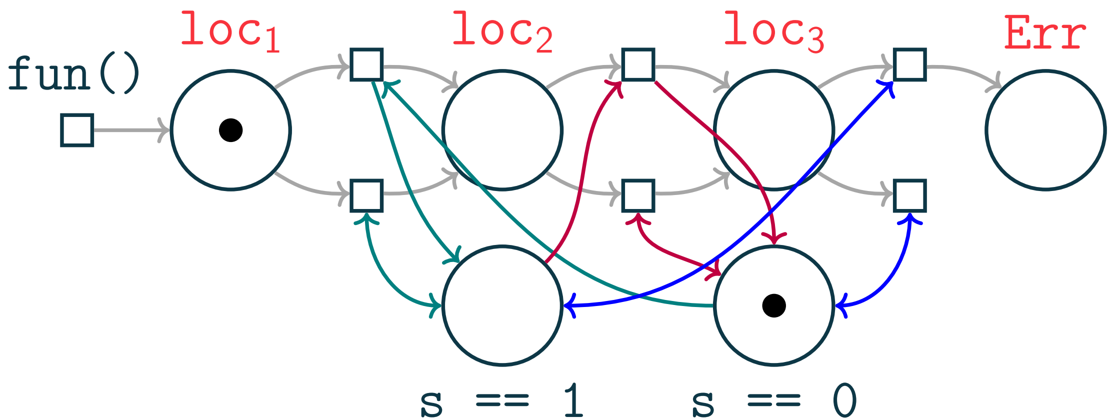

Can there be at least one token in `Err`?

$\Rightarrow$ Coverability problem!

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Program Synthesis [Feng et al., POPL 2017]

Synthesize a function:

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Area rotate(Area area,
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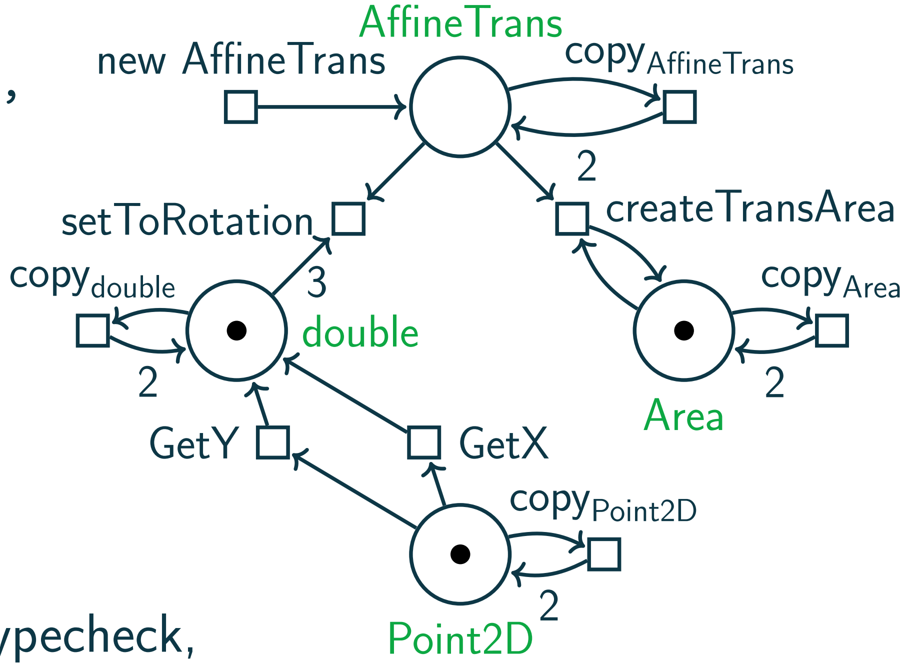

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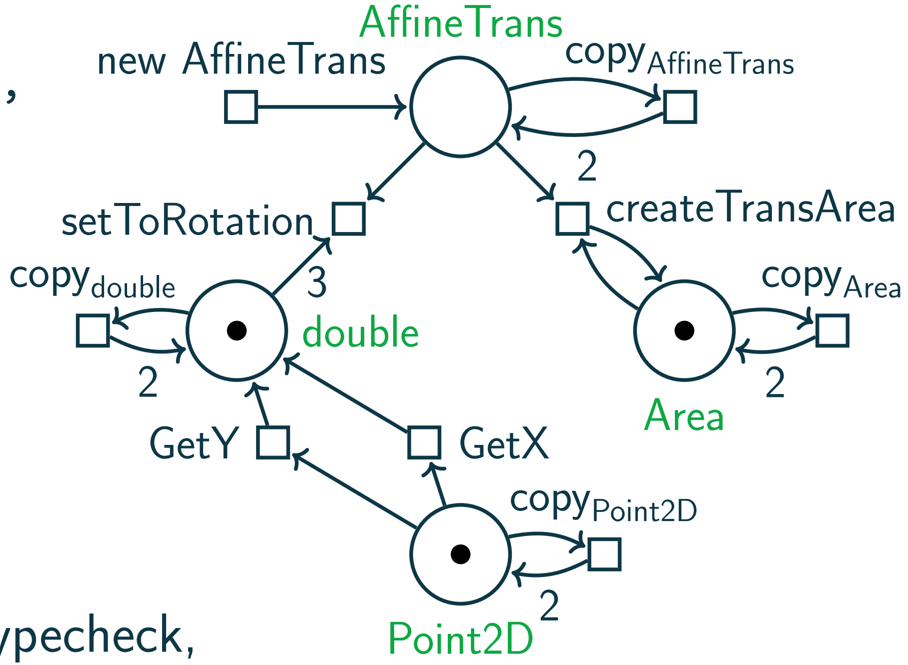

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$\Rightarrow$ Typechecking programs correspond to runs starting with a token in `Area`, `Point2D` and `double`, ending with exactly one token in `Area`

Some Applications of Petri Nets

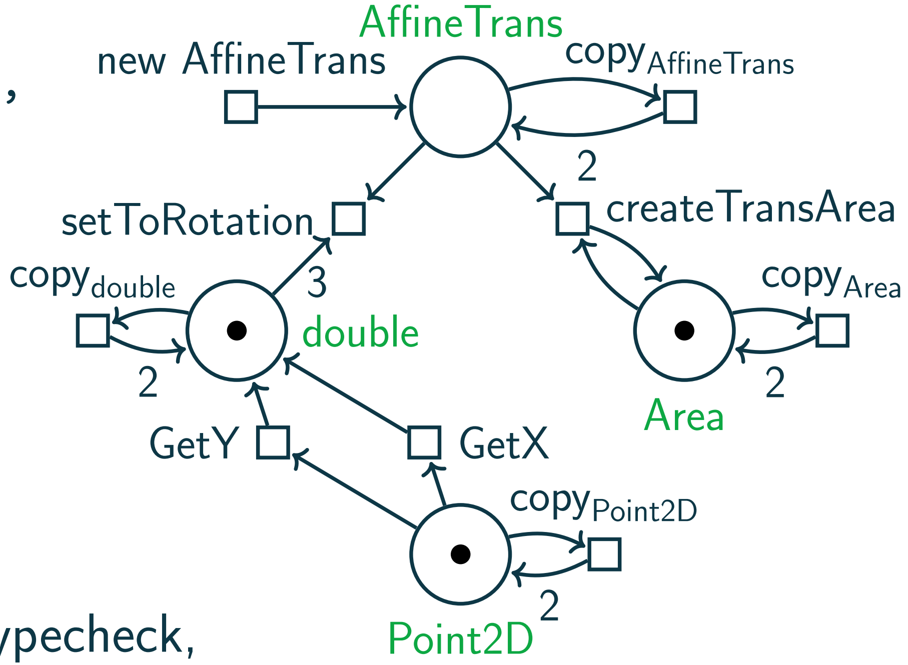
Program Synthesis [Feng et al., POPL 2017]

Synthesize a function:

```
Area rotate(Area area,
    Point2D point,
    double angle)
```

Use methods from the `java.awt.geom` library

java.awt.geom
`new AffineTrans()`
`double Point2D.getX()`
`double Point2D.getY()`
`void AffineTrans.setToRotation(double, double, double)`
`Area Area.createTransArea(AffineTrans)`



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$\Rightarrow$ Reachability Problem!

Some Applications of Petri Nets

More applications

- Scheduling
- Business Processes
- Chemical reaction networks
- . . .

Some Applications of Petri Nets

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- Scheduling
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- ...

Common theme: Short witnesses!

Short bug traces = easier to fix

Short synthesized programs = easier to understand

...

State of the Art

Coverability

- [Karp and Miller, 1967]: Karp-Miller trees

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- QCover [Blondin et al., 2016]: Backward algorithm with pruning, based on Continuous Petri Nets (tighter approximation)

State of the Art

Reachability

- [Kosaraju, 1982]: Complete algorithm for reachability, Ackermannian complexity

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- LoLA: Depth-first search, Random walks
- Issue: Few benchmarks for reachability with infinite state spaces
- MIST: Standard benchmark suite, but almost no reachability

Part II

Reachability Overapproximations

Approximations

Two sources of hardness in Petri Nets

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- Token counts must be integers

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- Token counts must be integers
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Relaxing either restriction gives us
an **overapproximation** of reachability

If a target is unreachable in the overapproximation, then it is unreachable in the Petri Net!

[Esparza et al., 2014], [Blondin et al., 2016]

Approximations

Continuous Petri Nets/Continuous token counts

Allow firing transitions by a fraction $\beta \in (0, 1]$

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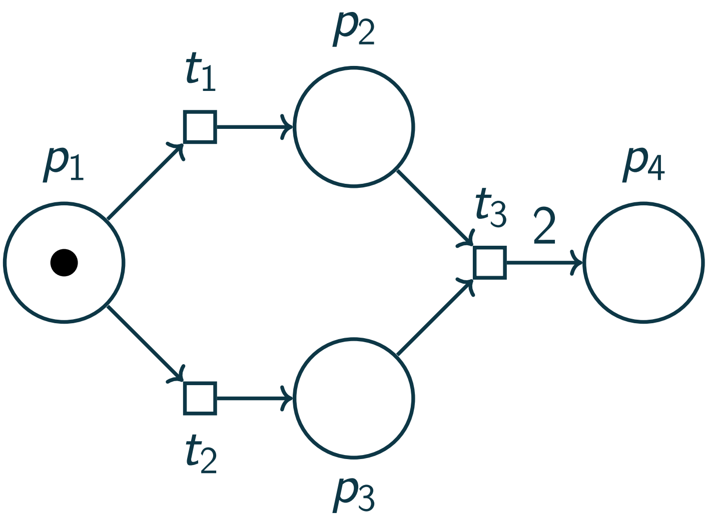

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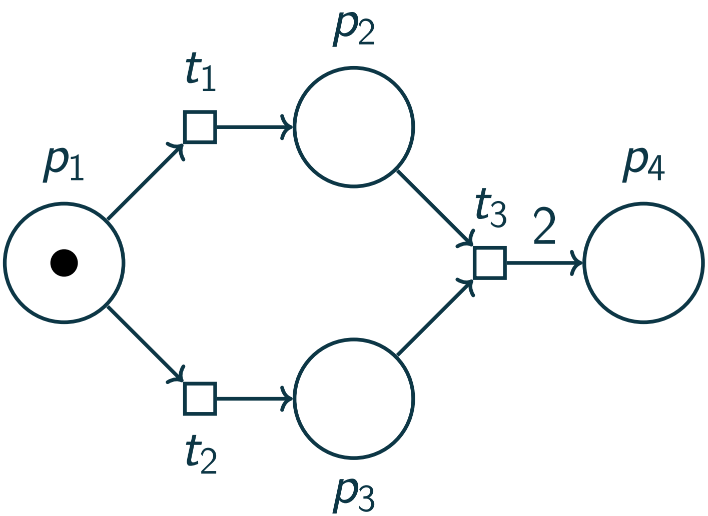

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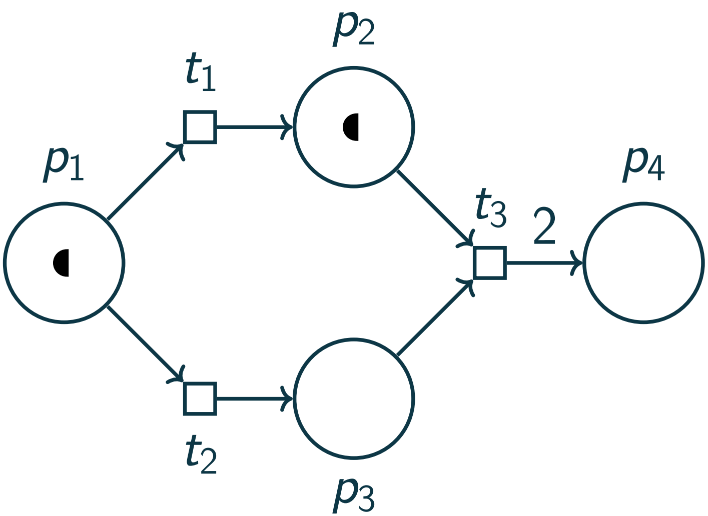

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$$(1, 0, 0, 0)$$
$$\xrightarrow{0.5 t_1} (0.5, 0.5, 0, 0)$$

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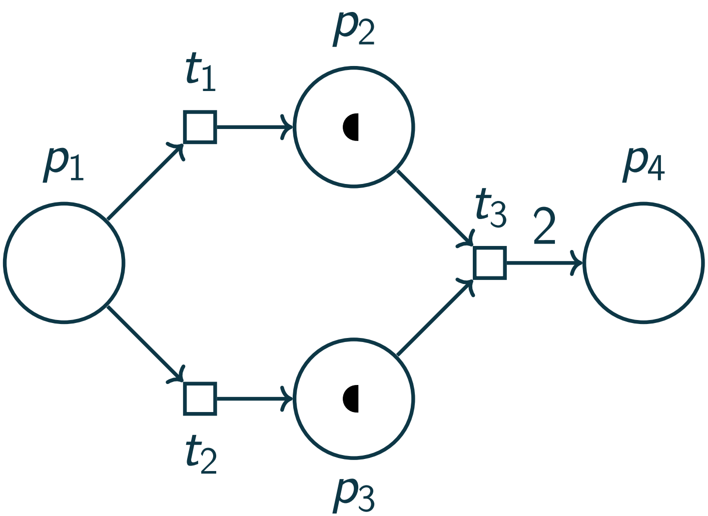

$$(1, 0, 0, 0) \xrightarrow{0.5t_1} (0.5, 0.5, 0, 0) \xrightarrow{0.5t_2} (0, 0.5, 0.5, 0)$$

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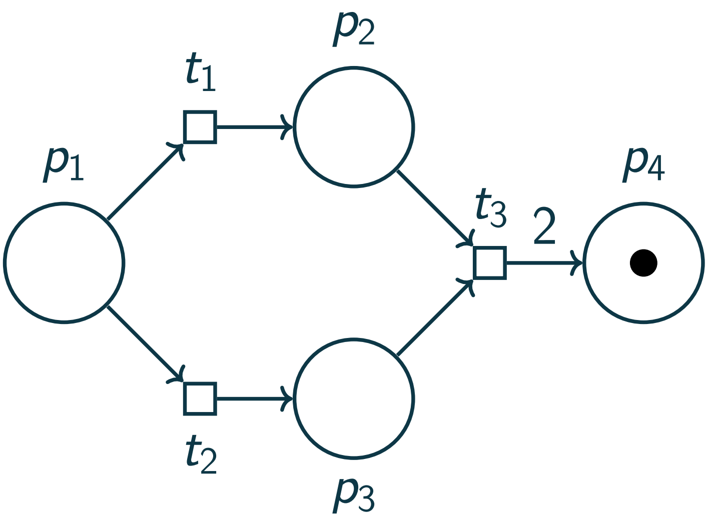

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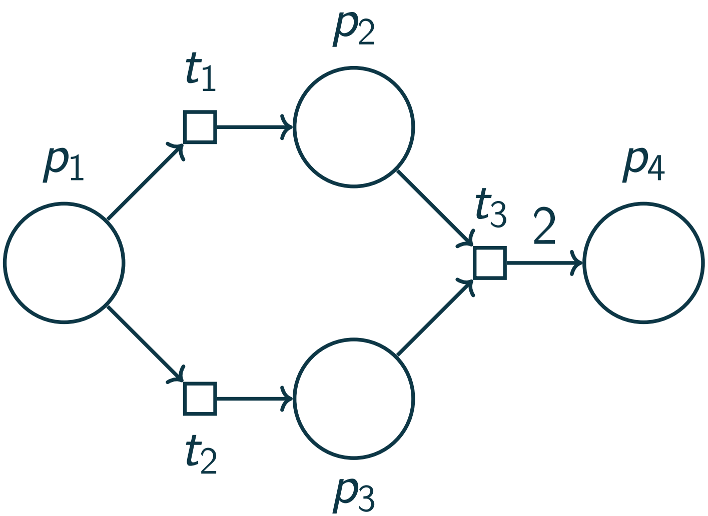

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Reachability is **Ptime-complete** [Fraca and Haddad, 2013]

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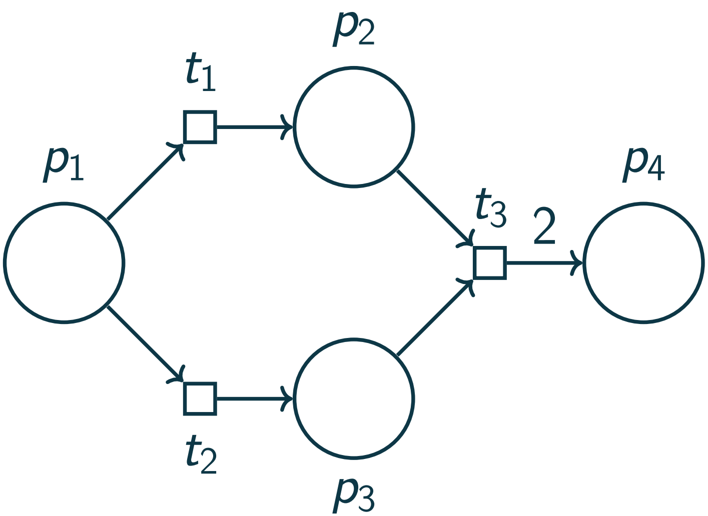

$$
\begin{aligned}
&(1, 0, 0, 0) \\
&\xrightarrow{0.5 t_1} (0.5, 0.5, 0, 0) \\
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&\xrightarrow{0.5 t_3} (0, 0, 0, 1)
\end{aligned}
$$

Reachability is **Ptime-complete** [Fraca and Haddad, 2013]

Alternatively, expressed as a formula

in existential $FO(\mathbb{Q}, +, <)$ — Satisfiability Modulo Theories/SMT

SMT Solving is fast in practice, e.g., via Z3 [Blondin et al., 2016]

Approximations

State Equation over $\mathbb{N}$/Negative token counts

Allow firing transitions when it would yield negative tokens

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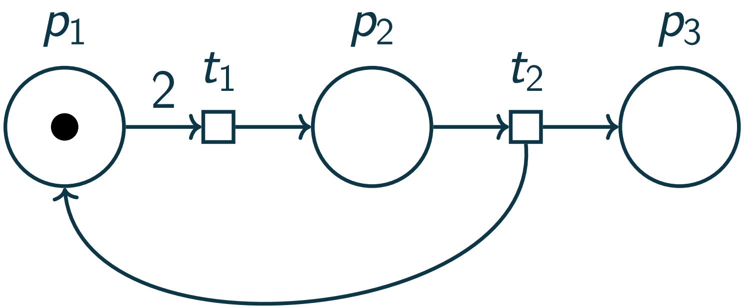

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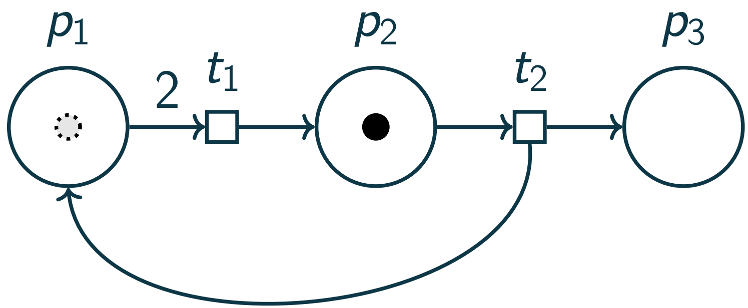

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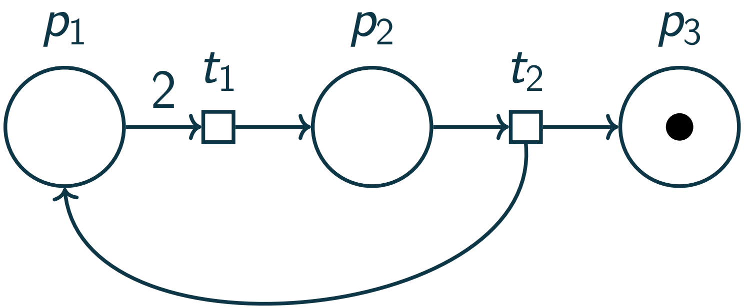

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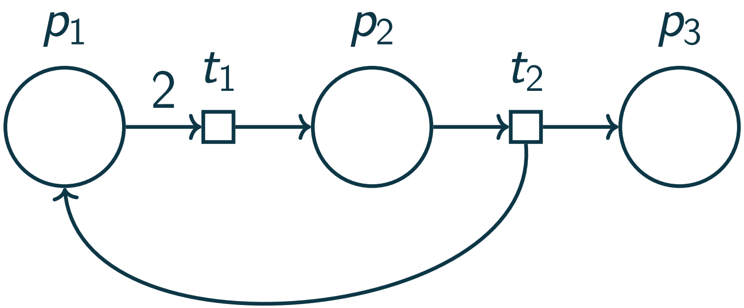

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$(1, 0, 0)$
$\xrightarrow{t_1} (-1, 1, 0)$
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Reachability from $(p_{1\,init}, p_{2\,init}, p_{3\,init})$ to $(p_{1\,final}, p_{2\,final}, p_{3\,final})$ if and only if $\exists t_1, t_2 \in \mathbb{N}$ such that:

$$p_{1\,final} = p_{1\,init} - 2 \cdot t_1 + t_2$$
$$p_{2\,final} = p_{2\,init} + t_1 - t_2$$
$$p_{3\,final} = p_{3\,init} + t_2$$

$\Rightarrow$ State Equation over $\mathbb{N}$

Approximations

State Equation over $\mathbb{N}$/Negative token counts

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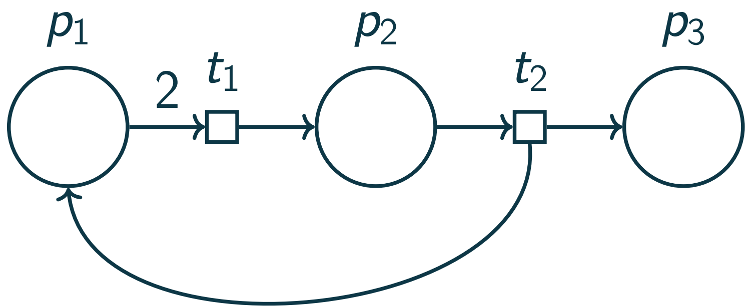

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$$p_{2\,final} = p_{2\,init} + t_1 - t_2 \qquad \Rightarrow \text{State Equation over } \mathbb{N}$$
$$p_{3\,final} = p_{3\,init} + t_2$$

Solved via Integer Linear Programming (ILP)

$\Rightarrow$ Computable in NP.

Approximations

State Equation over $\mathbb{Q}$/Continuous, negative token counts

Again amounts to solving the State Equation, but over $\mathbb{Q}$.

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State Equation over $\mathbb{Q}$/Continuous, negative token counts

Again amounts to solving the State Equation, but over $\mathbb{Q}$.

Solved via Linear Programming (LP)
$\Rightarrow$ Computable in Ptime.

Approximations

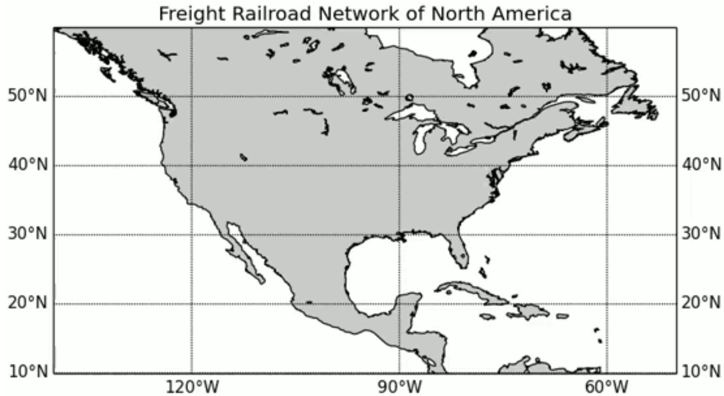
Overview

	Complexity	**Computed Via**
Petri Nets	Non-elementary	Kosaraju's
Continuous Petri Nets	Ptime-complete	SAT/SMT
State Equation over $\mathbb{N}$	NP-complete	Integer Lin. Prog.
State Equation over $\mathbb{Q}$	Ptime-complete	Lin. Prog.

Part III

Directed Search Algorithms

Directed Search Algorithms



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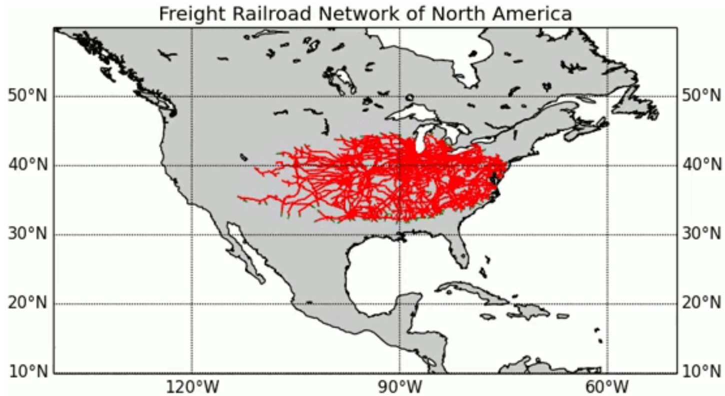

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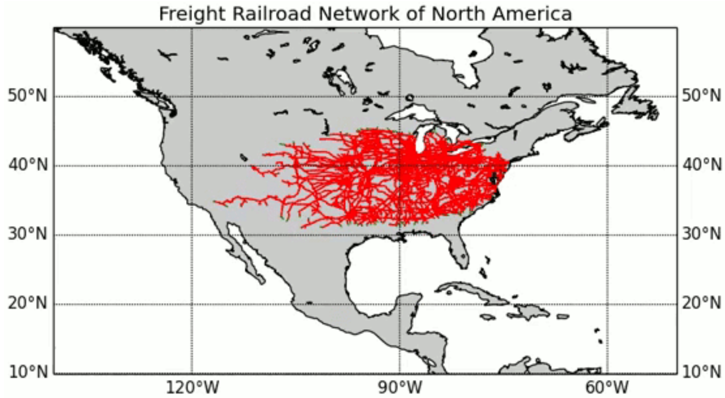

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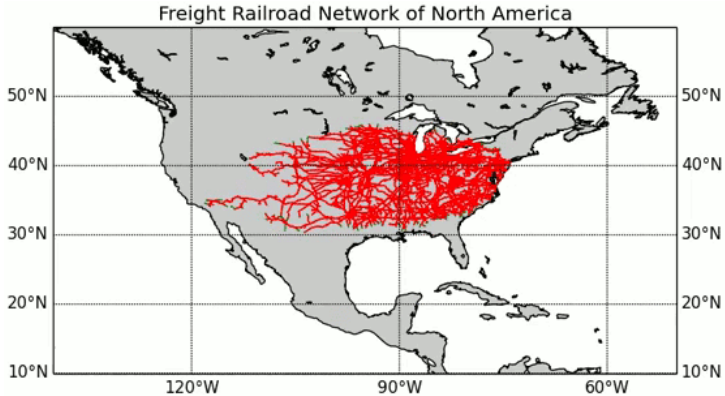

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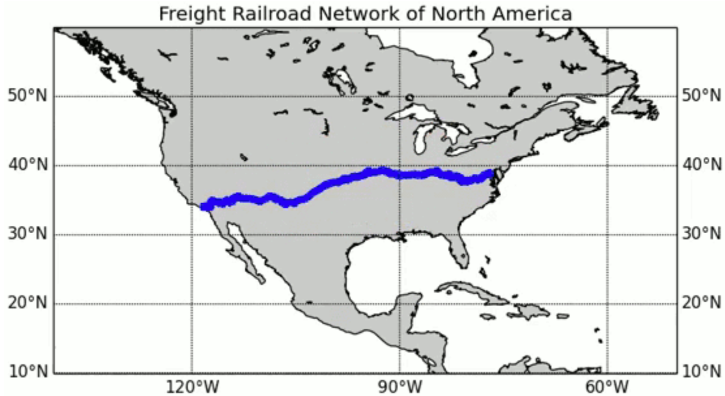

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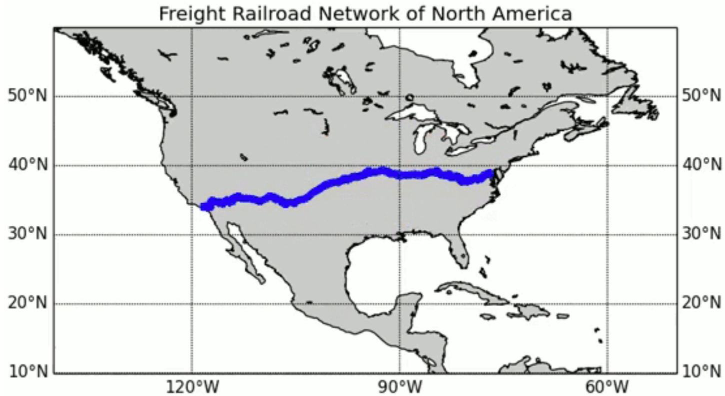

Directed Search Algorithms can handle very large graphs

Used successfully in AI, network routing, . . .

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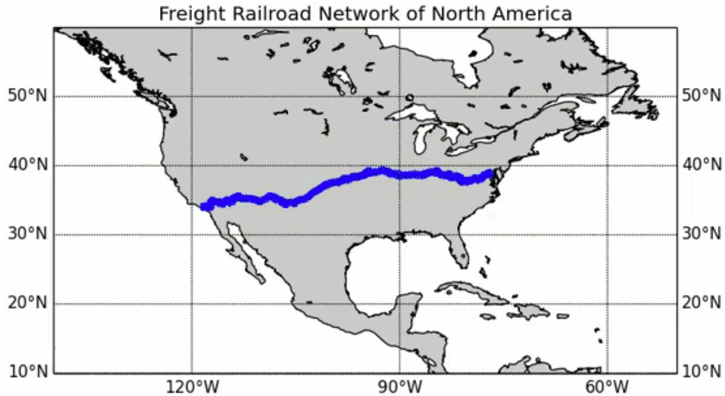

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Petri Nets have (infinite) reachability graphs!

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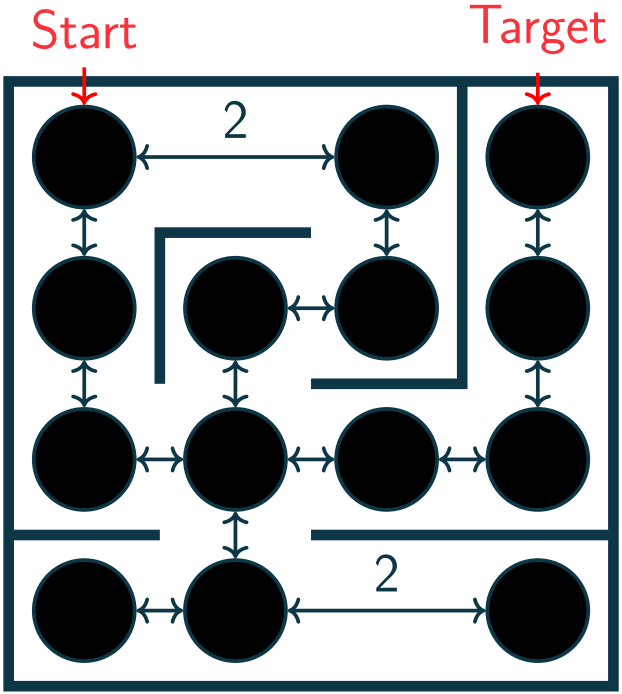
First: Refresher on Directed Search Algorithms

Afterwards: How to apply them to Petri nets

Directed Search Algorithms
Dijkstra's/Breadth First Search



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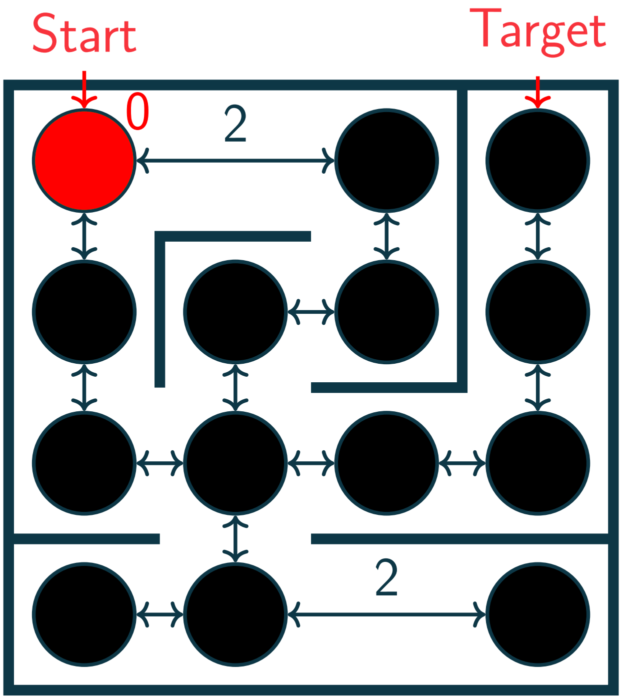
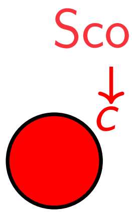




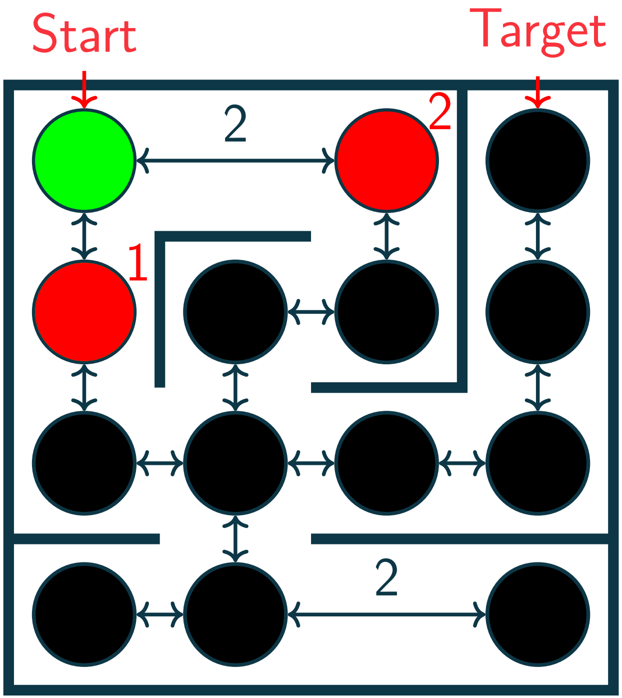
Frontier nodes:
Next nodes to explore

Explore node with
lowest score in frontier

Dijkstra's:
Score = Distance from Start

Directed Search Algorithms

Dijkstra's/Breadth First Search



Score

c

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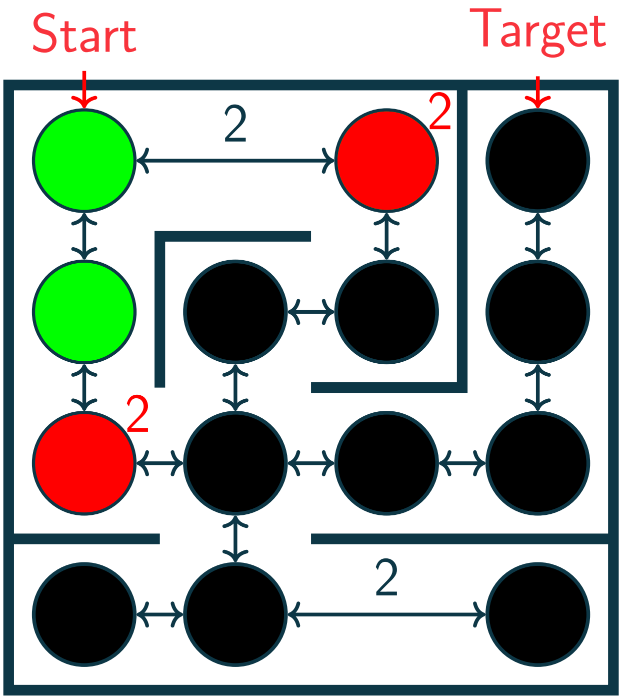

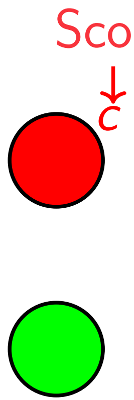

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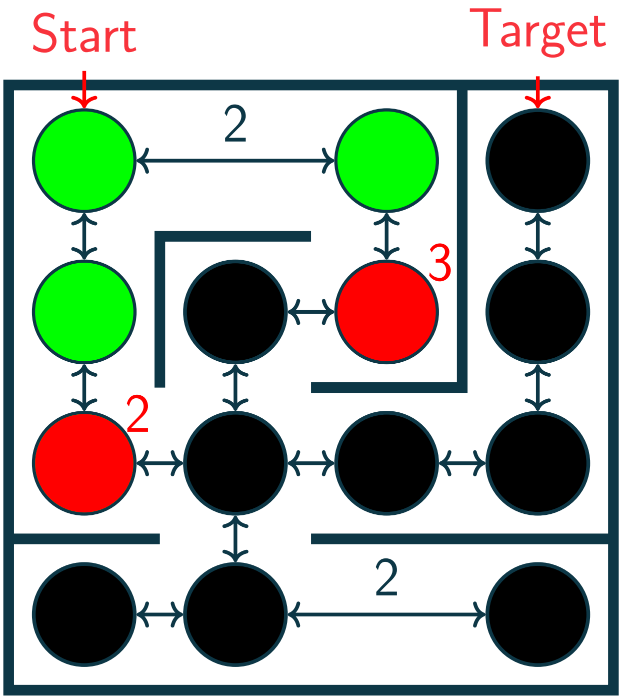

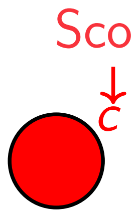

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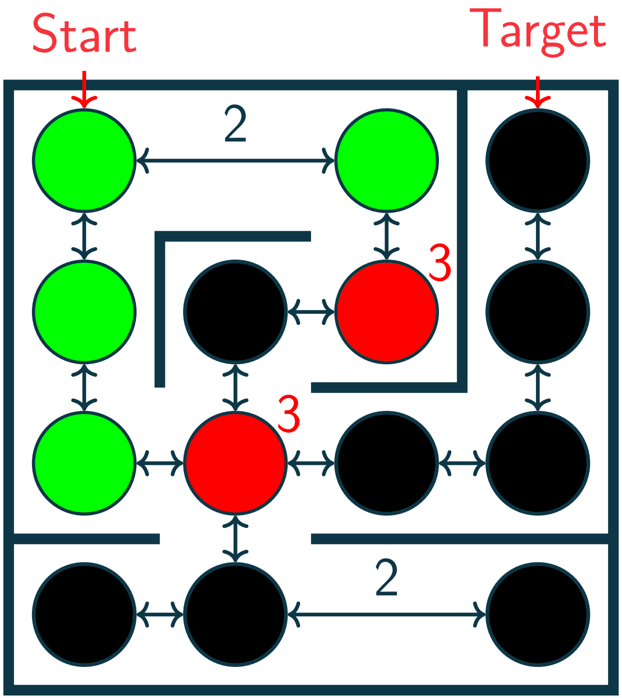

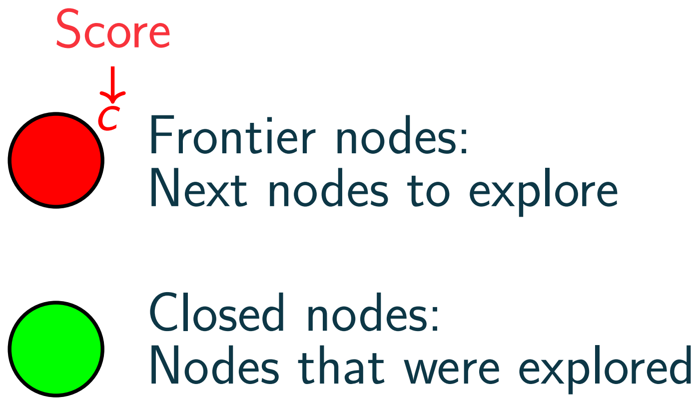

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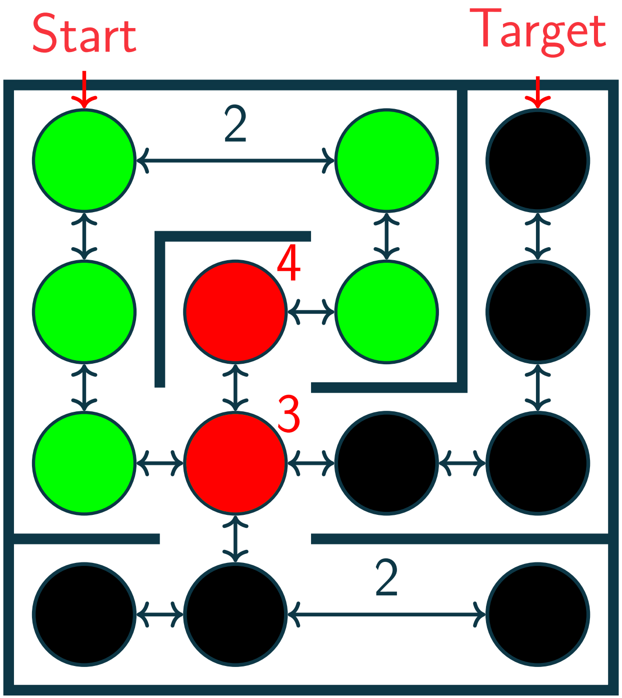

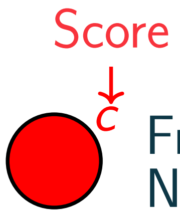

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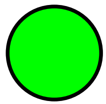

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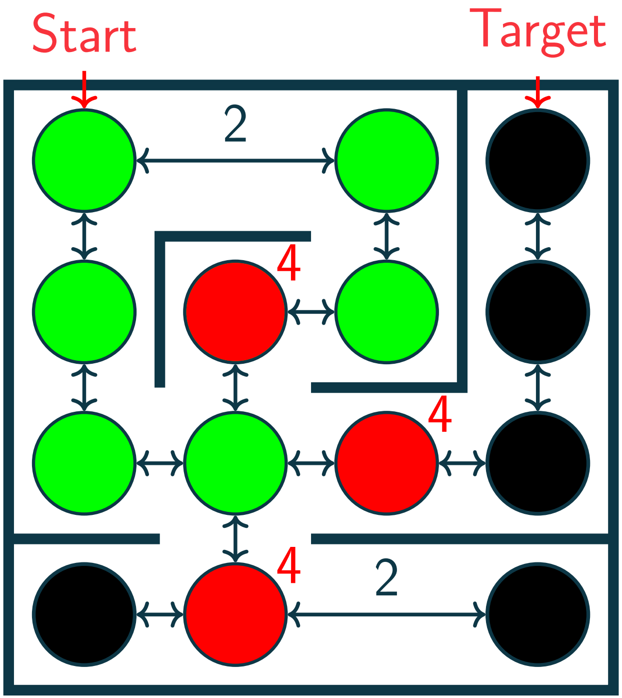

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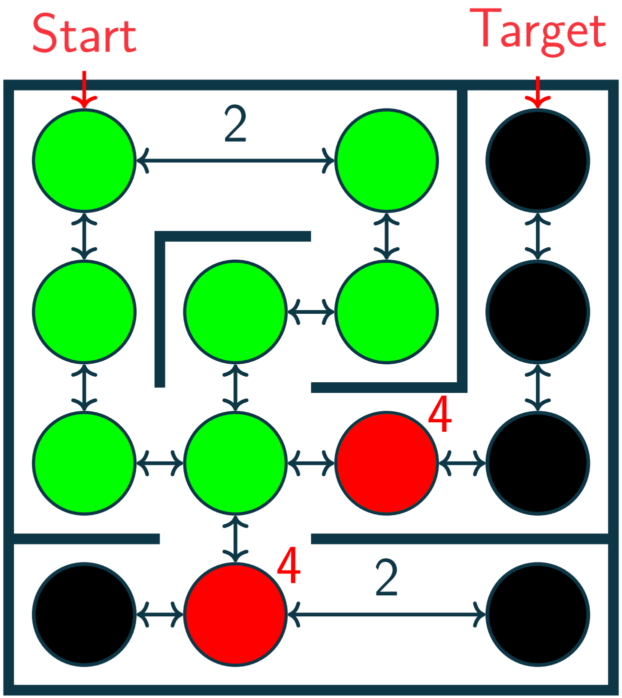

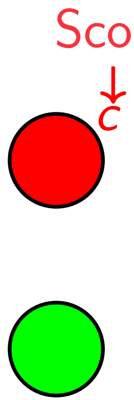

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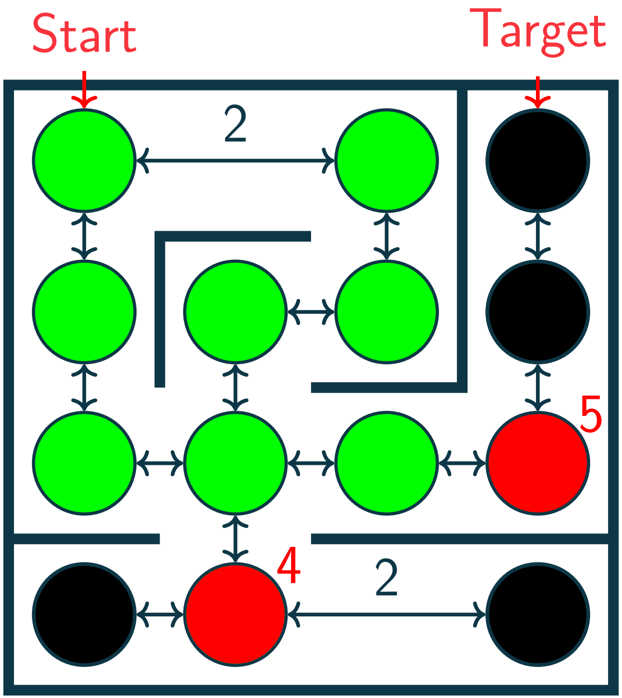

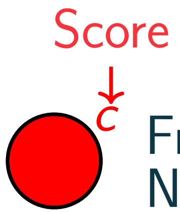

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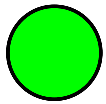

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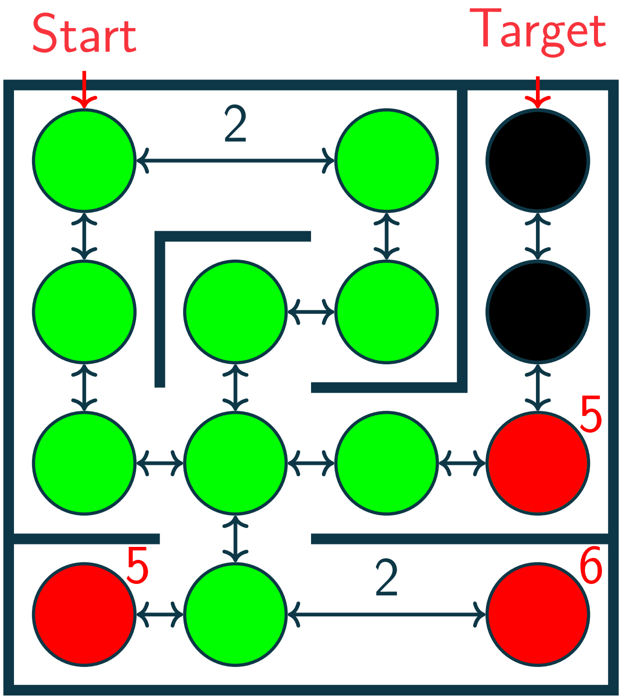
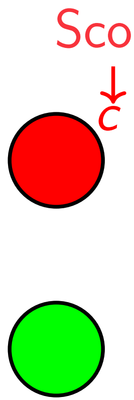
Directed Search Algorithms

Dijkstra's/Breadth First Search





Frontier nodes:
Next nodes to explore

Closed nodes:
Nodes that were explored

Explore node with
lowest score in frontier

Dijkstra's:
Score = Distance from Start

Directed Search Algorithms

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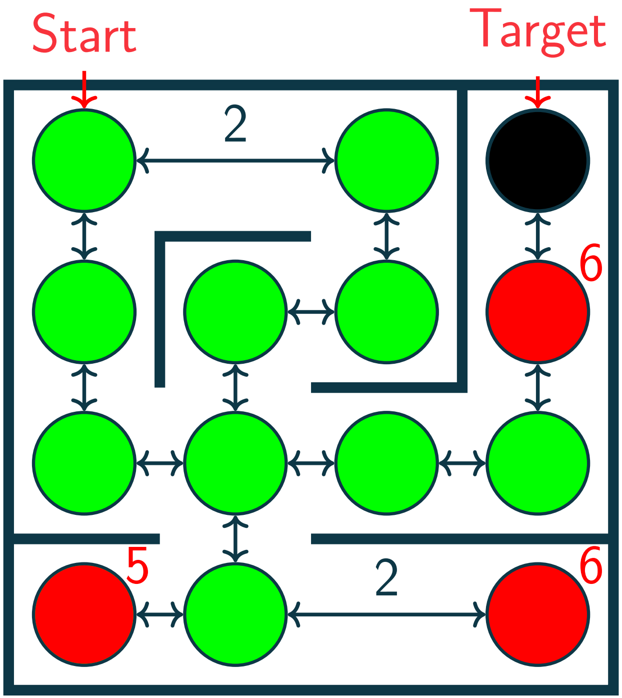

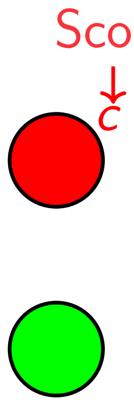

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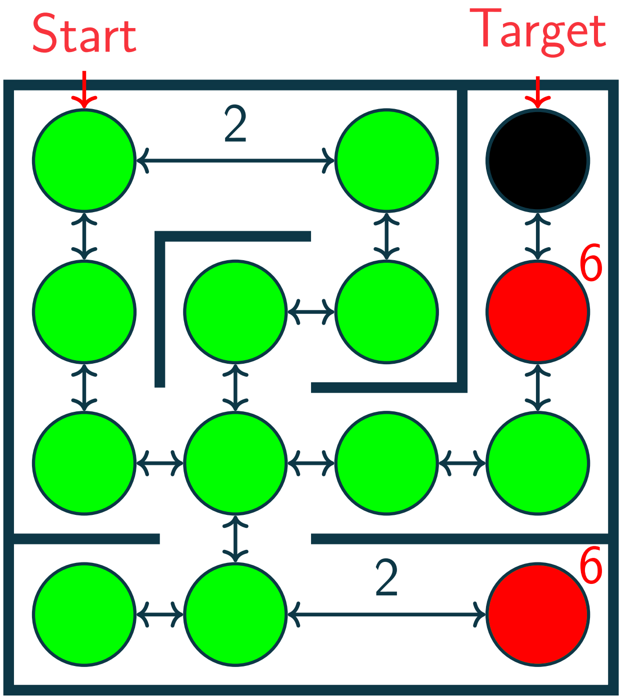

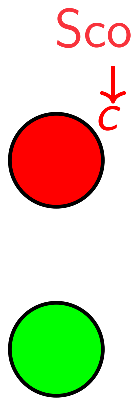

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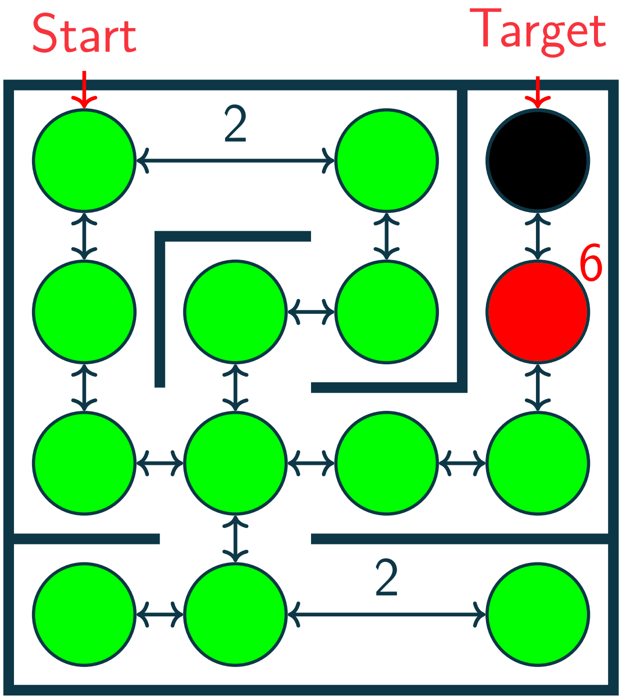

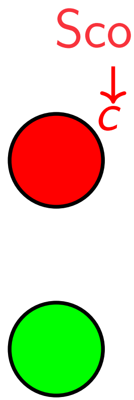

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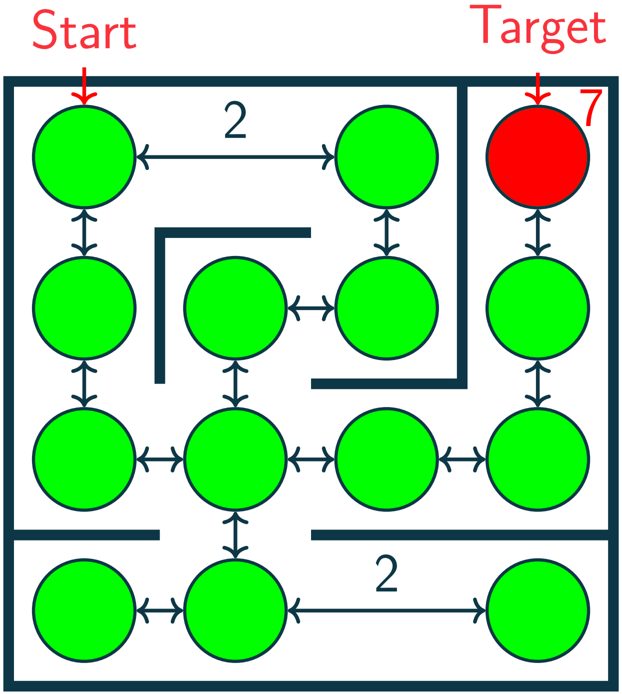

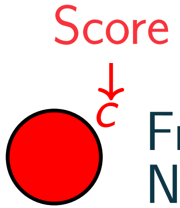

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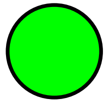

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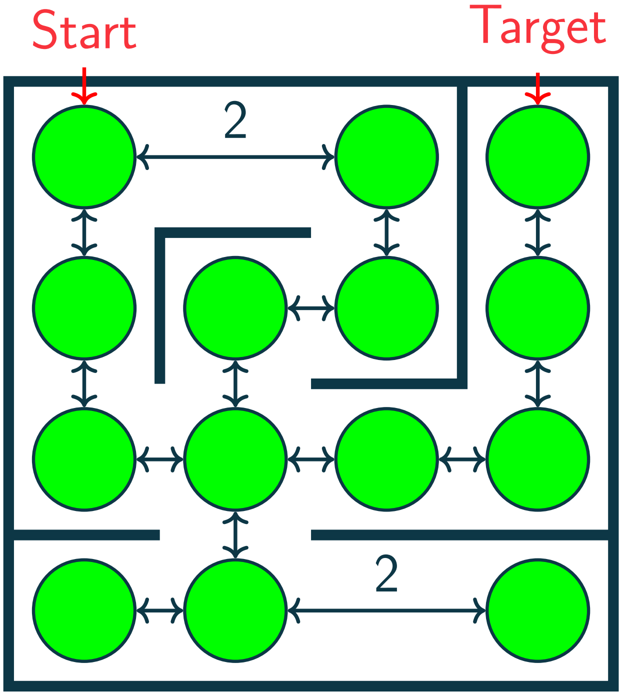

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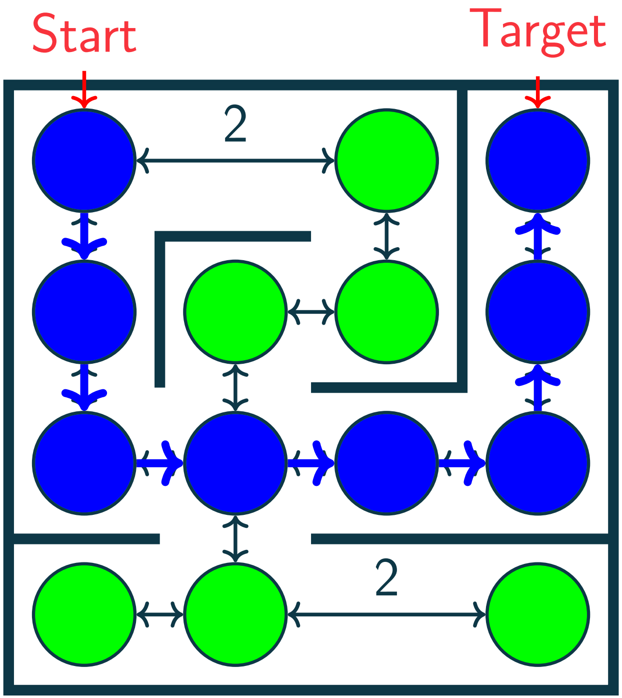

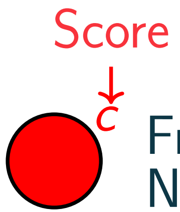

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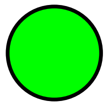

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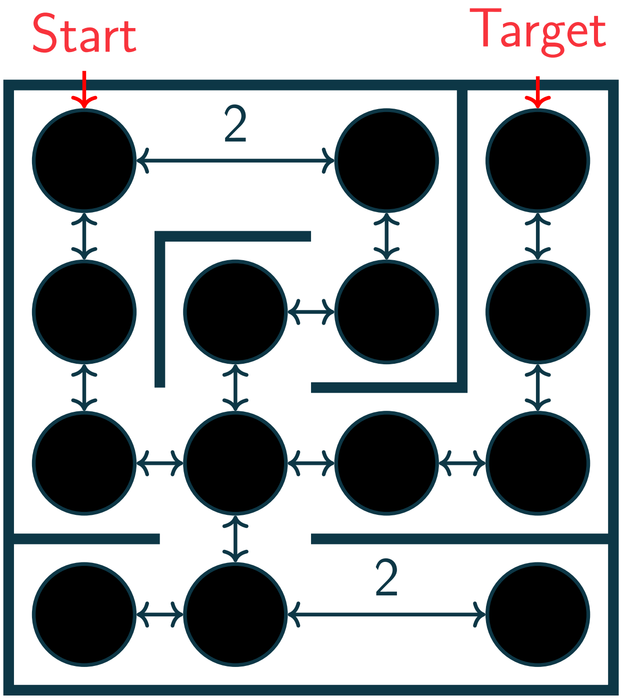
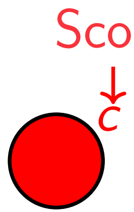
Directed Search Algorithms

Greedy Best-First Search (GBFS)



Heuristic estimation needed!

Heuristic: Grid-distance



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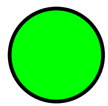

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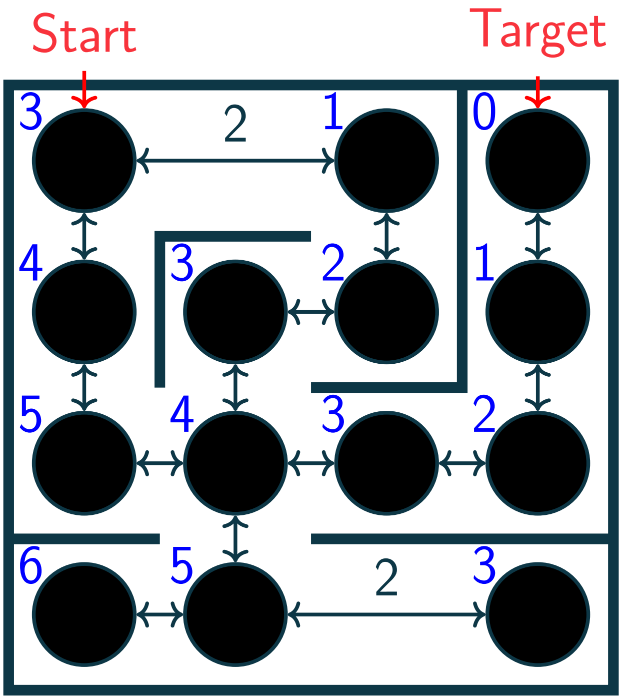
Directed Search Algorithms

Greedy Best-First Search (GBFS)



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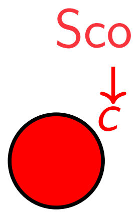

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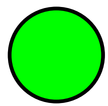

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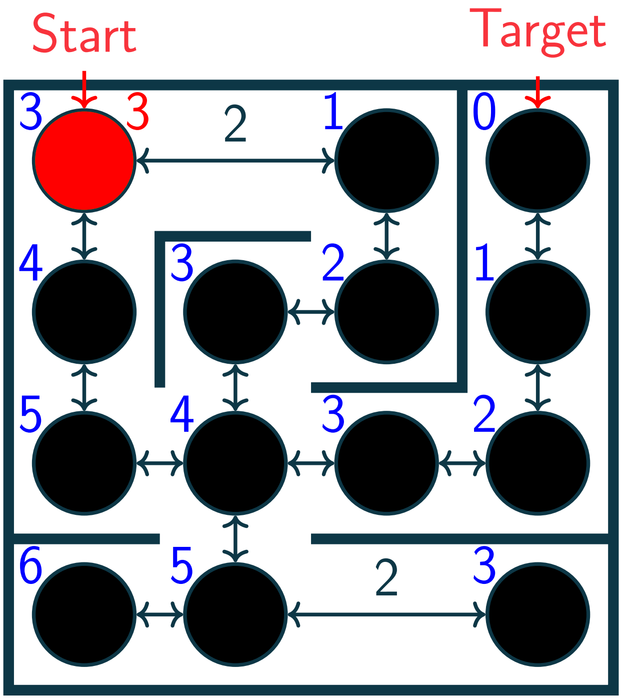

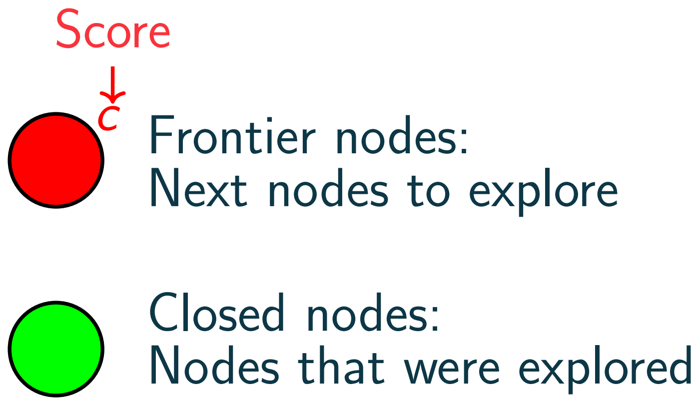

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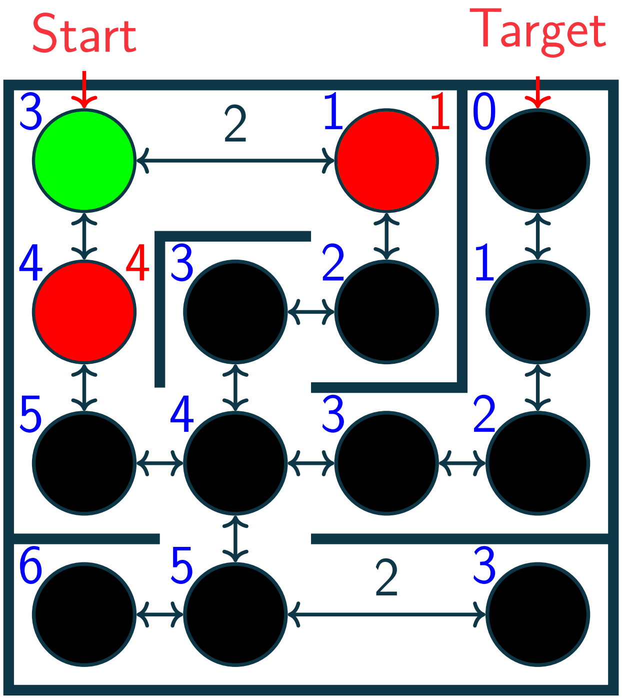

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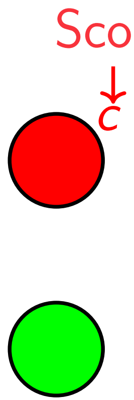

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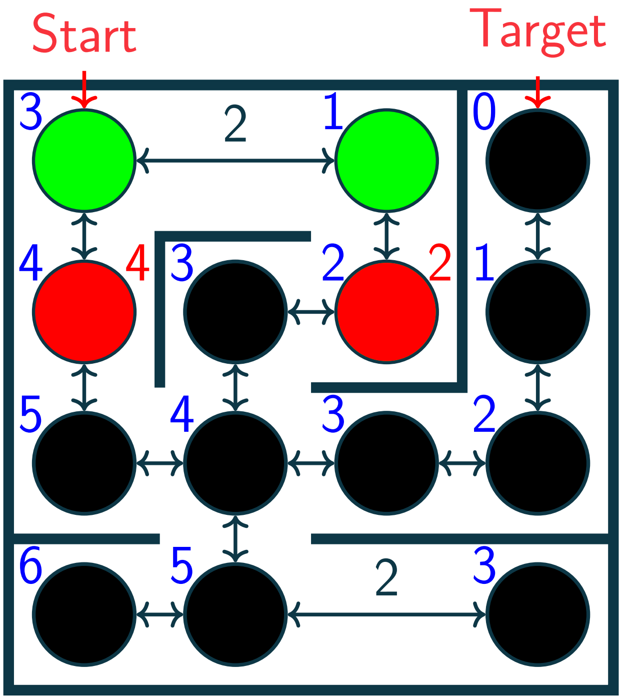

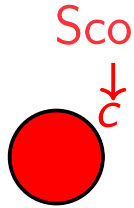

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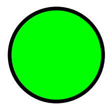

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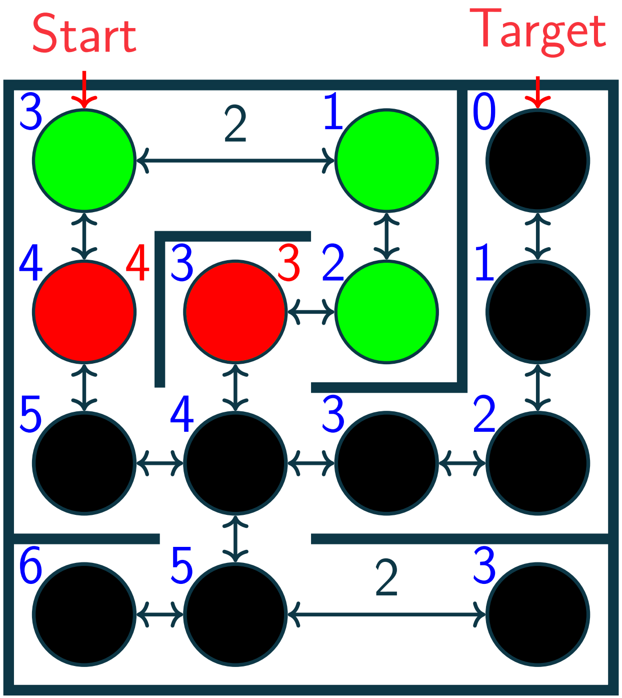

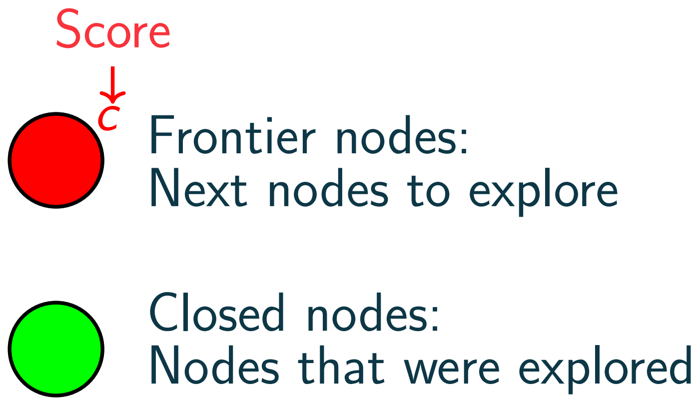

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Directed Search Algorithms

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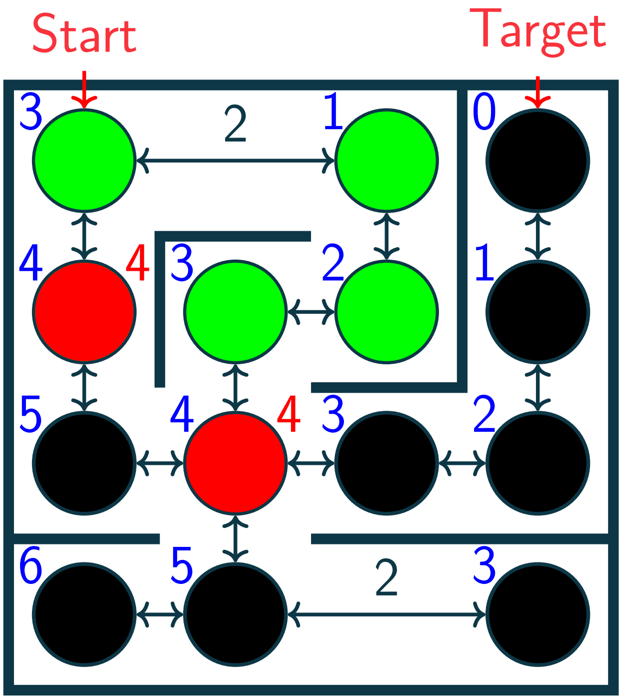

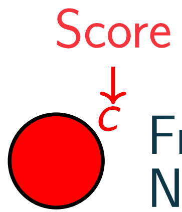

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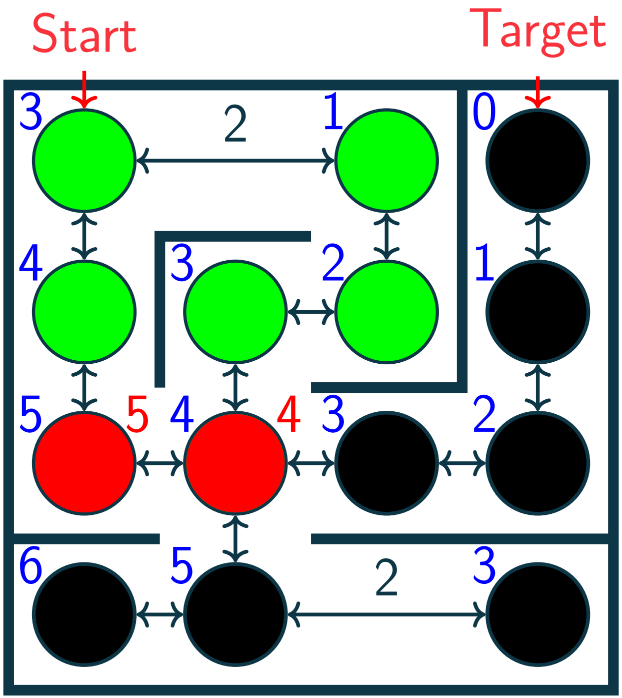

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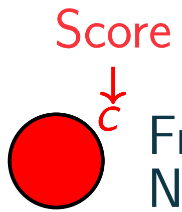

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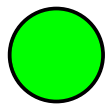

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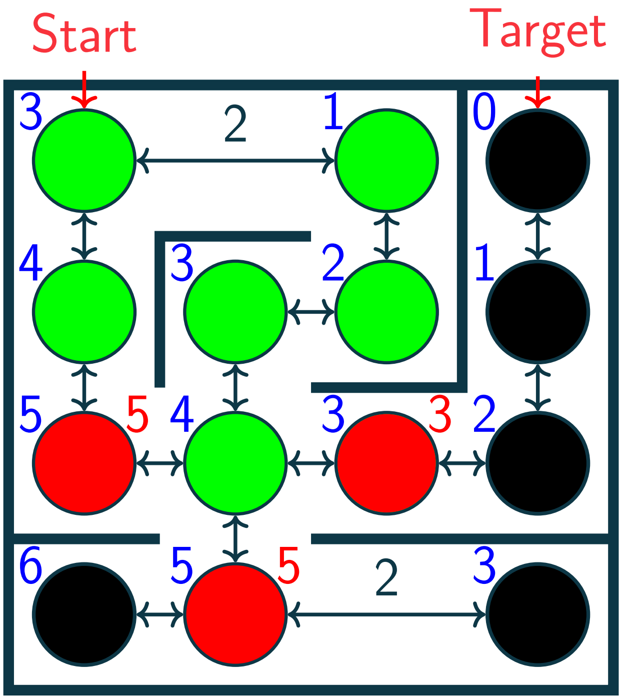

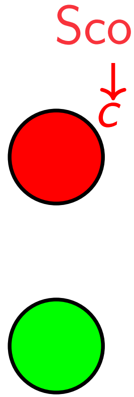

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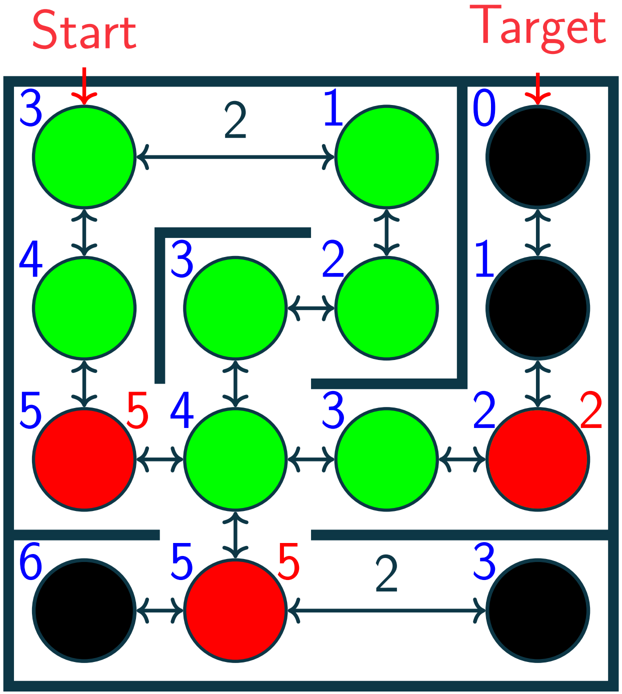

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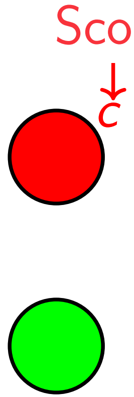

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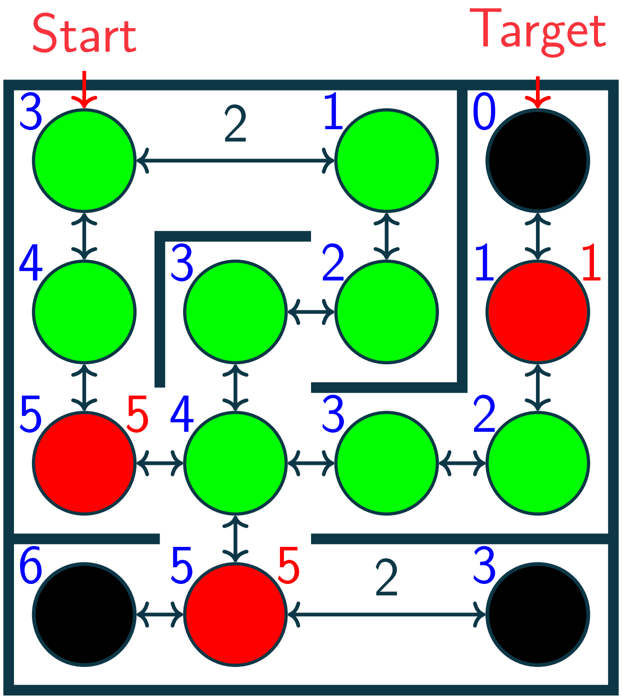

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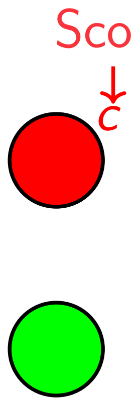

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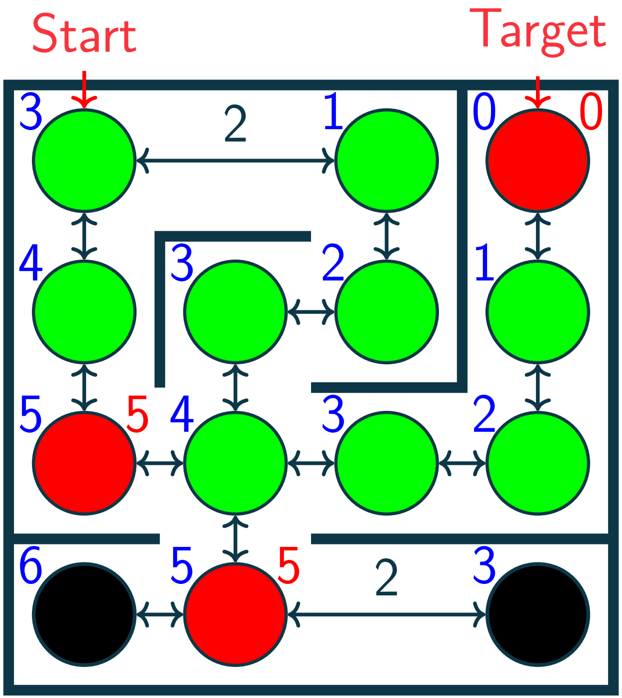

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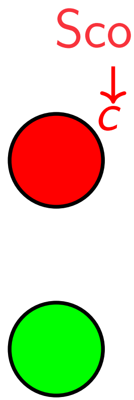

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Greedy Best-First Search:
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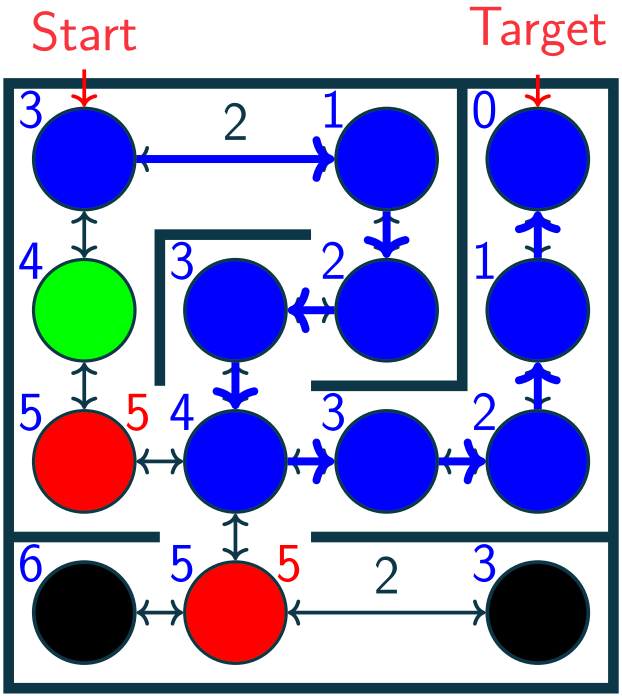
Directed Search Algorithms

Greedy Best-First Search (GBFS)



Heuristic estimation needed!

Heuristic: Grid-distance

Score

c

Frontier nodes:
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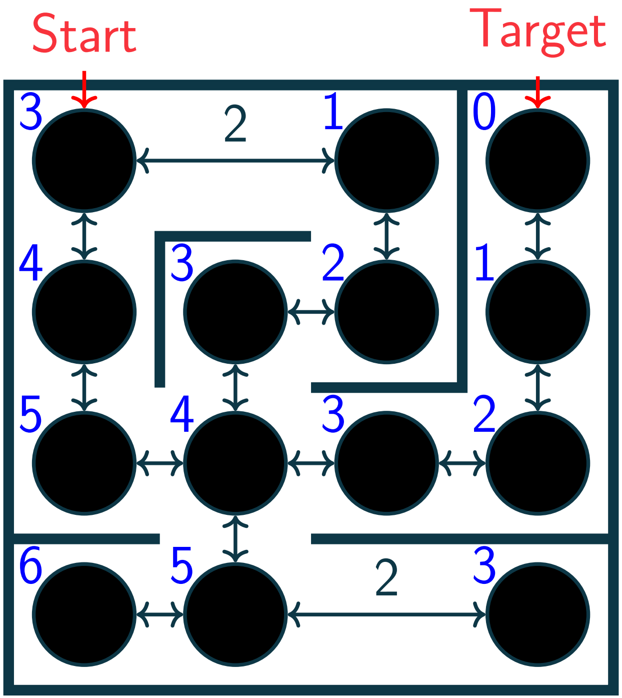
Greedy Best-First Search:
Score = Distance to Target

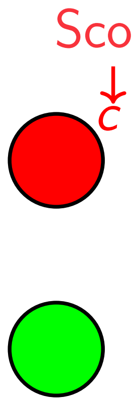
Directed Search Algorithms

A*



Heuristic estimation needed!

Heuristic here: Grid-distance



Frontier nodes:
Next nodes to explore

Closed nodes:
Nodes that were explored

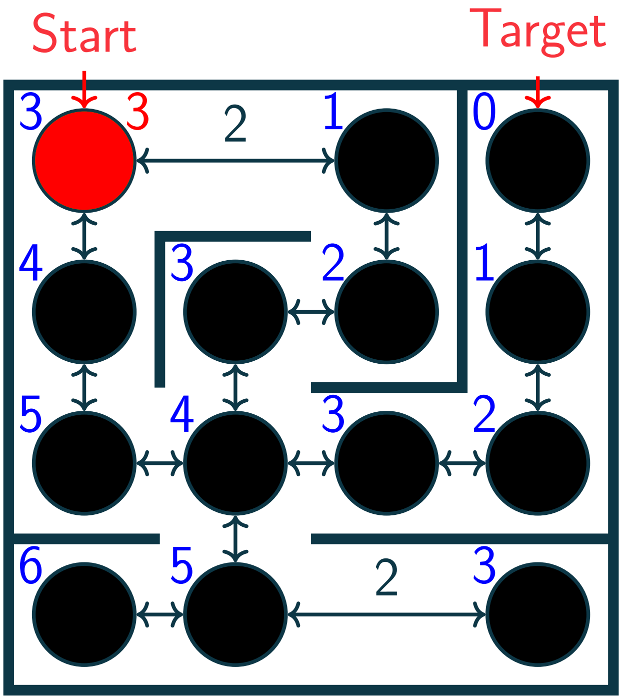
Explore node with
lowest score in frontier

A*:

Score = Distance from Start + Distance to Target

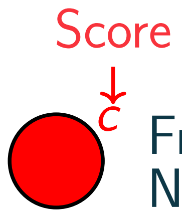
Directed Search Algorithms

A^*



Heuristic estimation needed!

Heuristic here: Grid-distance



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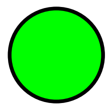

Closed nodes:
Nodes that were explored

Explore node with lowest score in frontier

A^*:

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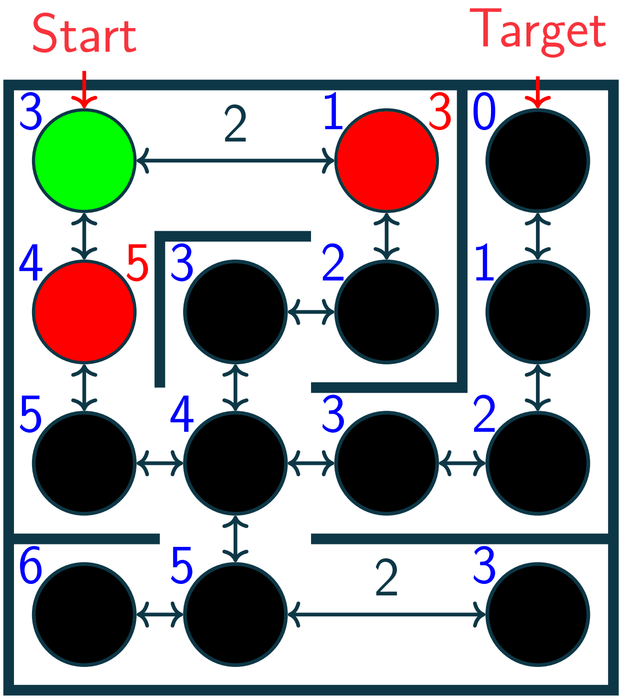
Directed Search Algorithms

A*



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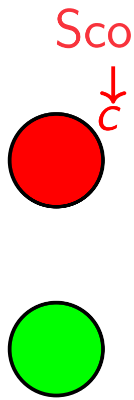

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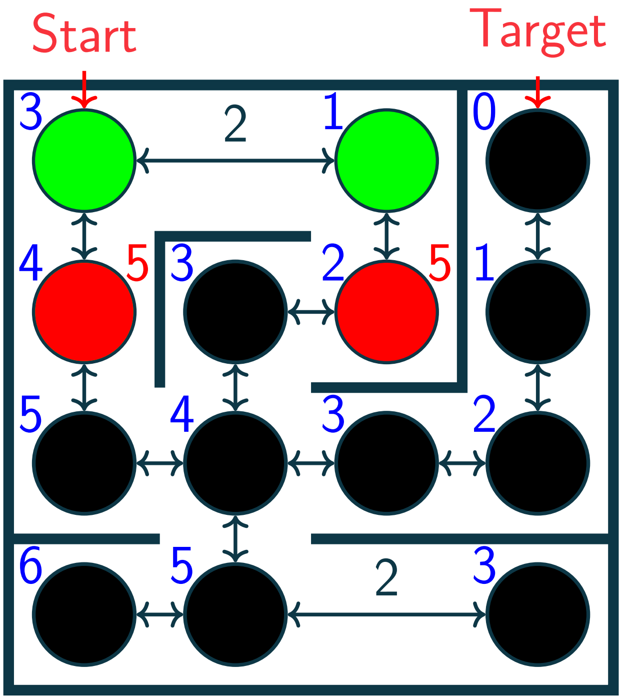
Directed Search Algorithms

A*



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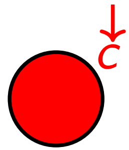

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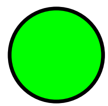

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Directed Search Algorithms

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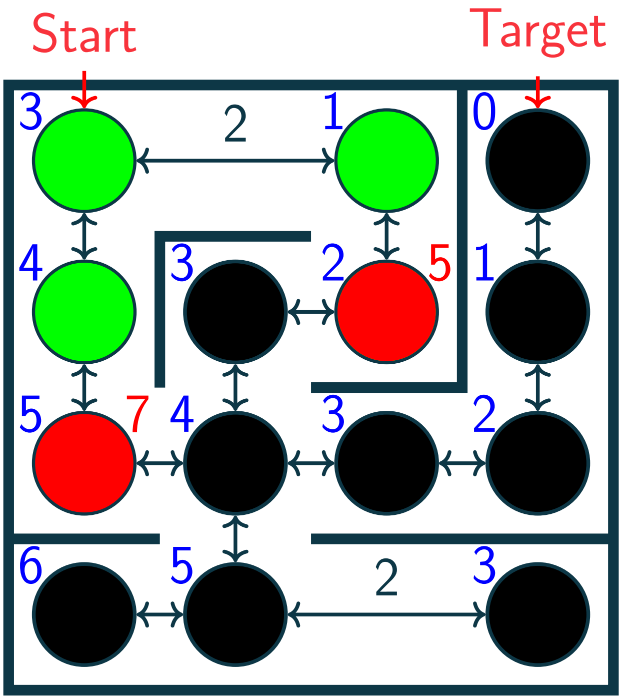

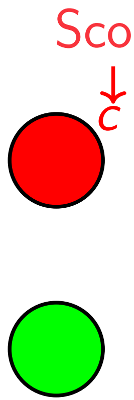

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Directed Search Algorithms

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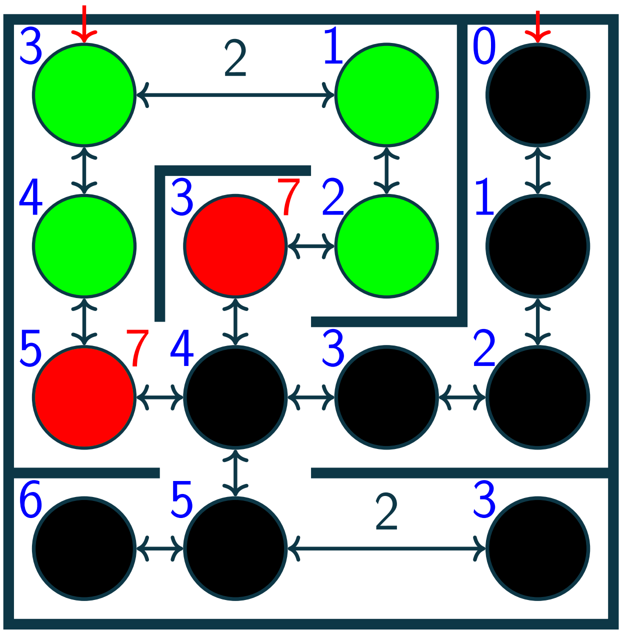

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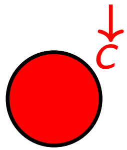

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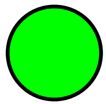

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Directed Search Algorithms

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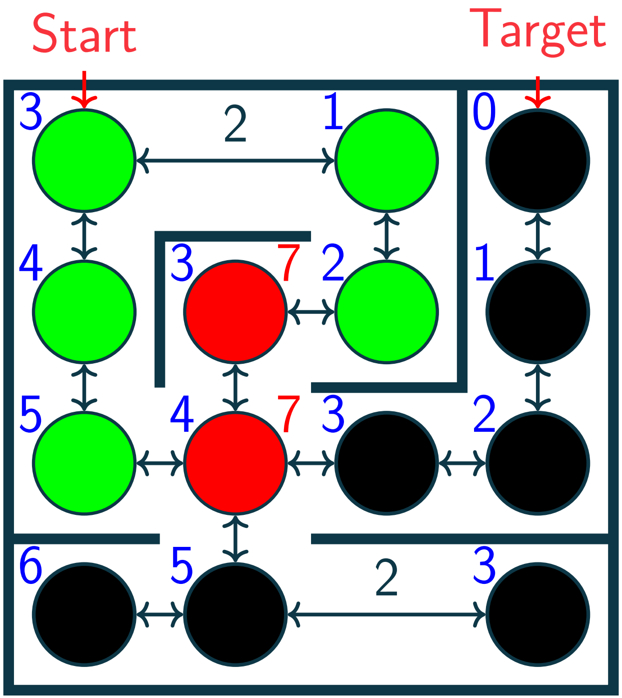

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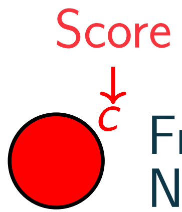

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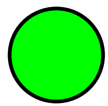

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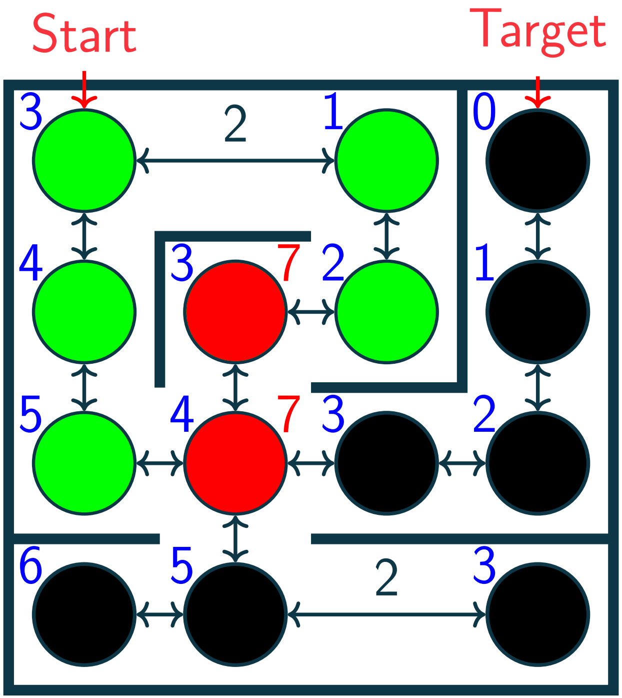

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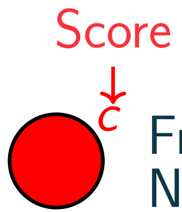

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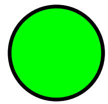

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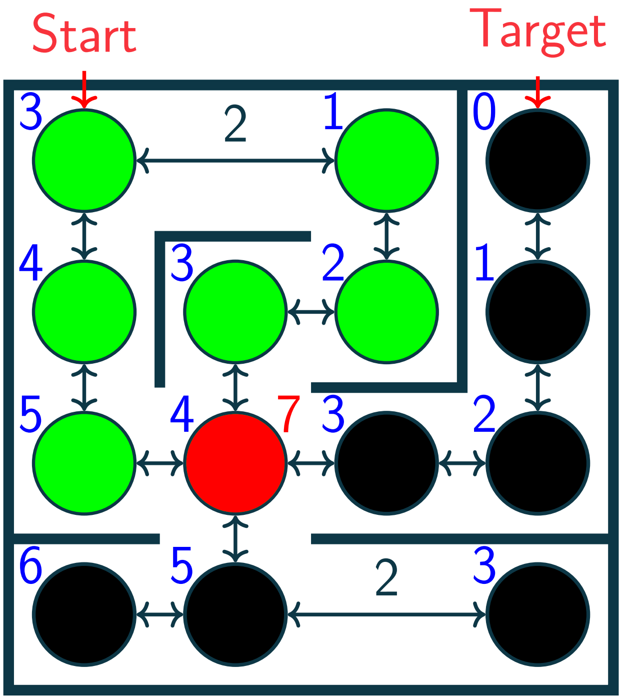

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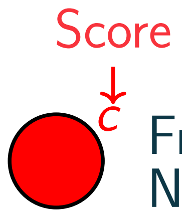

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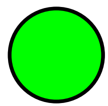

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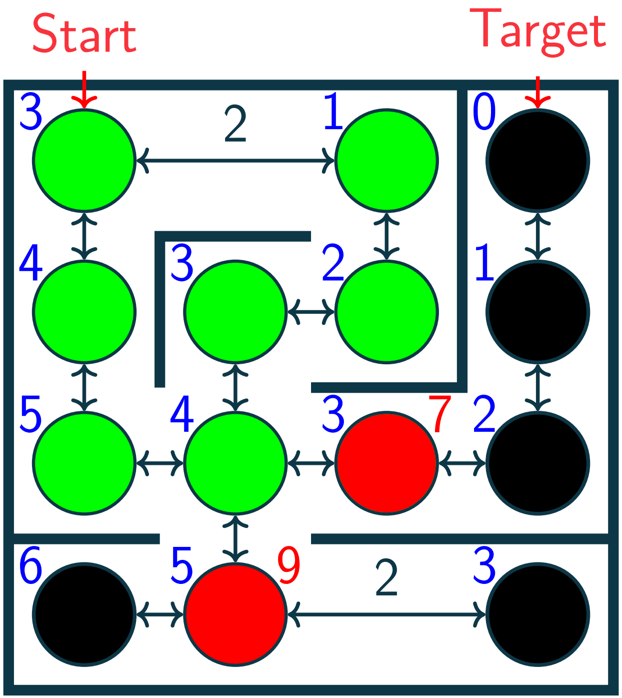

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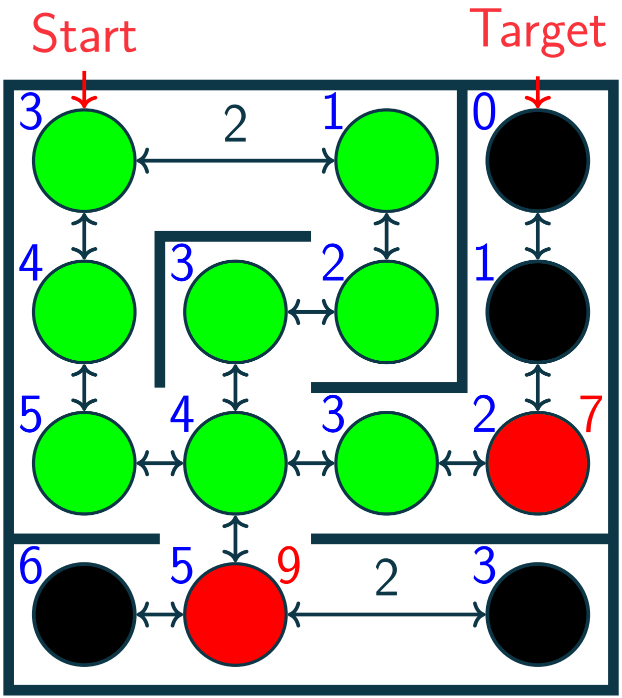

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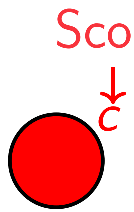

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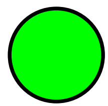

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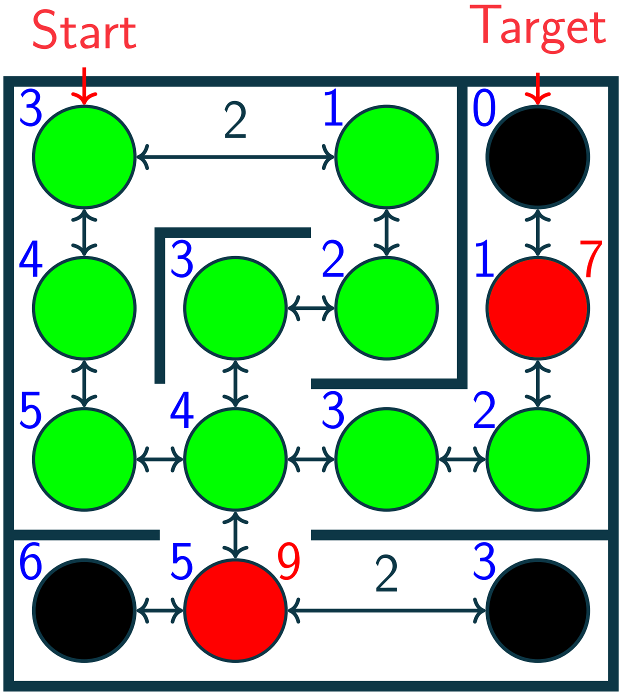

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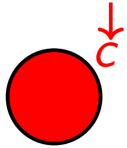

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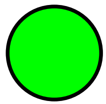

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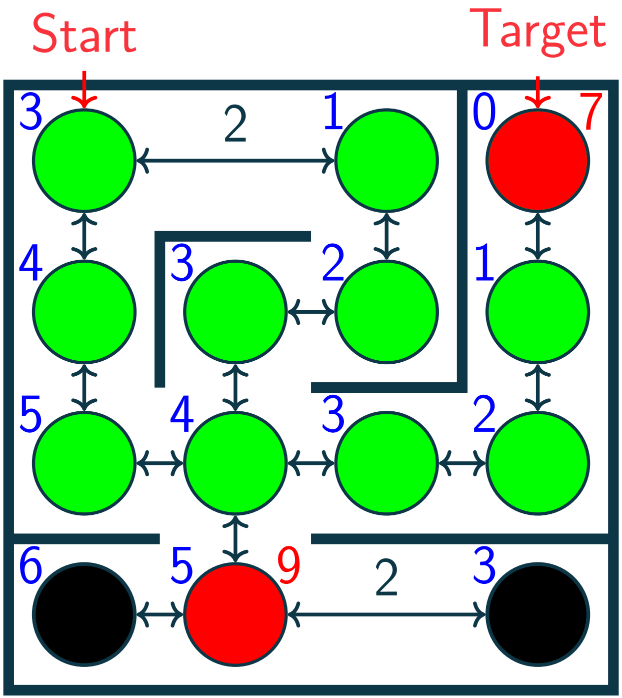

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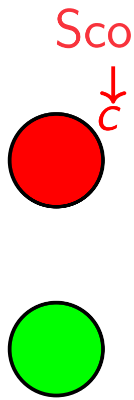

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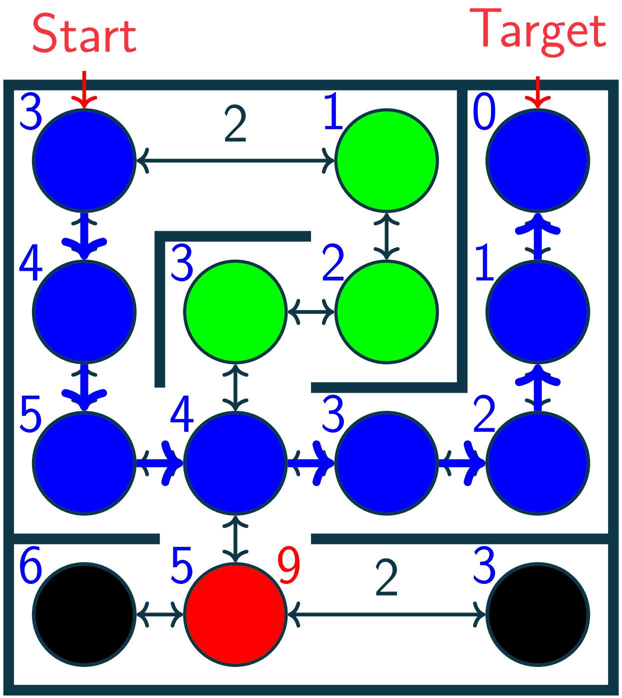

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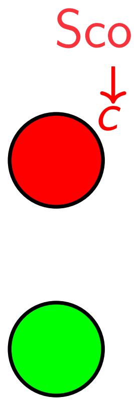

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$$\begin{tikzcd}
 & b \arrow[dr, "h(b)"] & \\
a \arrow[ur, "c(a,b)"] \arrow[rr, "h(a)"'] & & \text{target}
\end{tikzcd}$$

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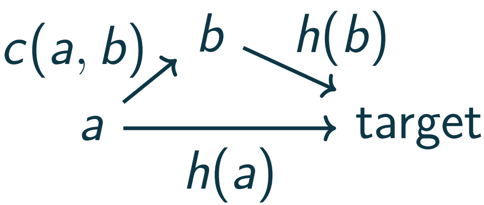

$\boxed{\text{Can we get better guarantees for GBFS?}}$

Termination for GBFS

Observation: Result from this work

GBFS terminates on infinite graphs with a reachable target
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Idea: GBFS cannot follow any infinite path forever without making progress towards the target

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How do we obtain heuristics from overapproximations?

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For any Petri net reachability overapproximation *approx*, h_{approx} is unbounded, admissible and consistent!

Applying Directed Search to Petri Nets

- Key insight: Modern ILP/SMT solvers allow computing shortest paths for reachability overapproximations fast
  $\Rightarrow$ ILP/SMT allow optimization of solutions

- Directed search based on reachability overapproximations gives formal guarantees:
  Shortest path (A*), Termination (GBFS)

- Highly efficient in practice $\Rightarrow$ rest of the talk

Part IV

Experimental Results

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Gurobi for (integer) linear programming,
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Reachability benchmarks: program synthesis,
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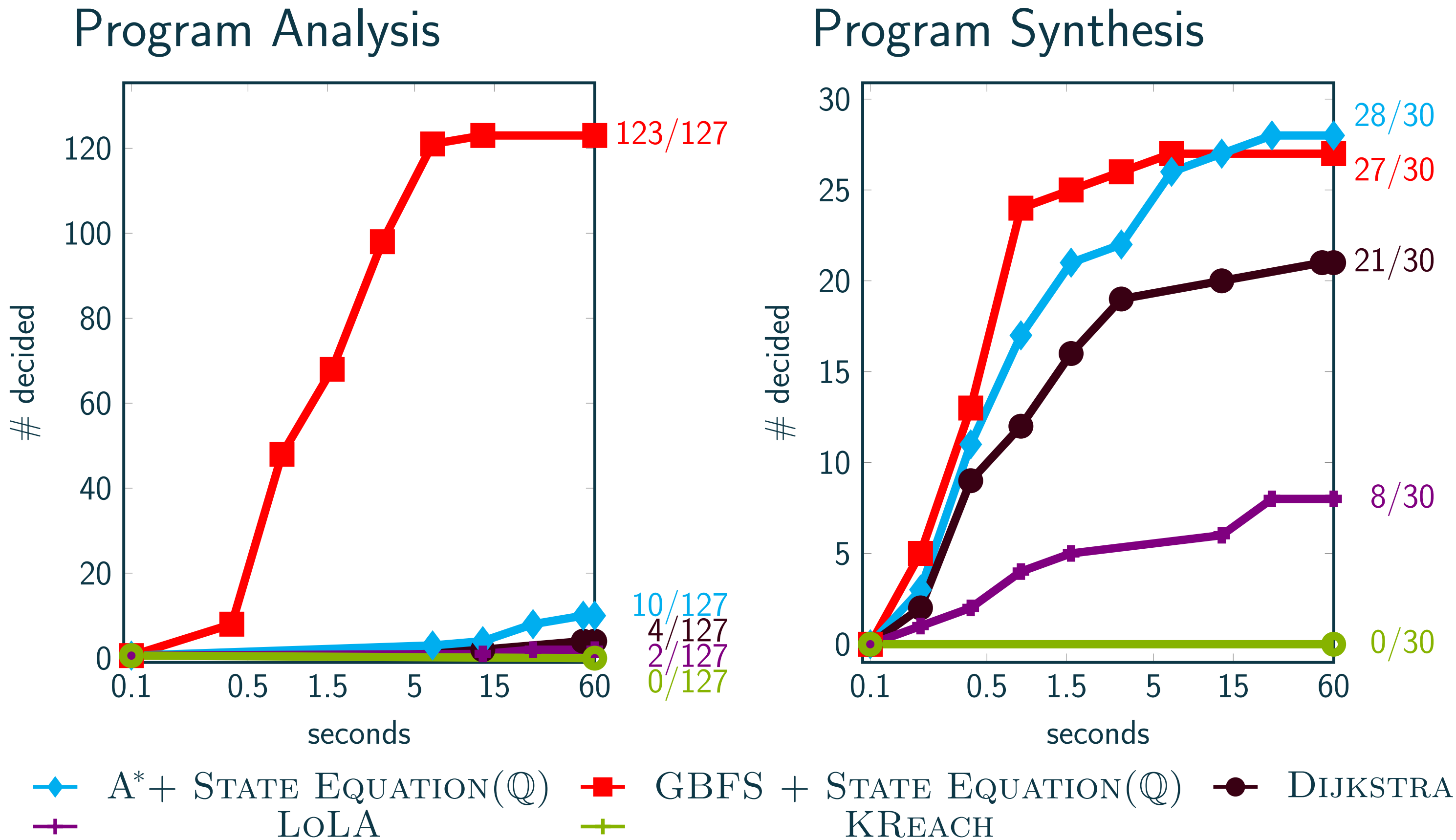
Focus is on reachable instances
(but exploratory results confirm known effectiveness of approximations for unreachable instances)

Experimental Results: Reachability

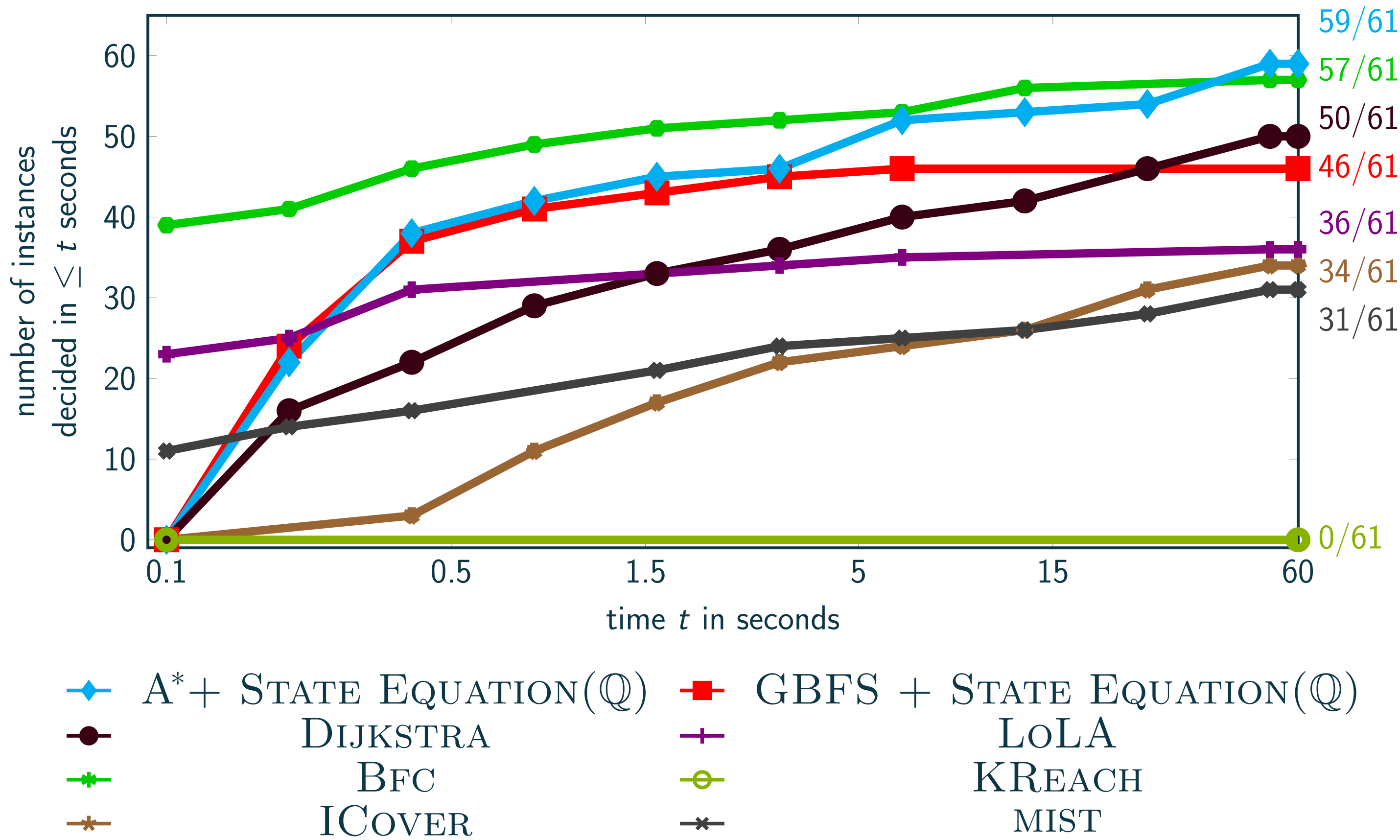


Guided Search outperforms existing approaches (by orders of magnitude)

Experimental Results: Coverability

Even competitive against dedicated coverability solvers

Program Analysis



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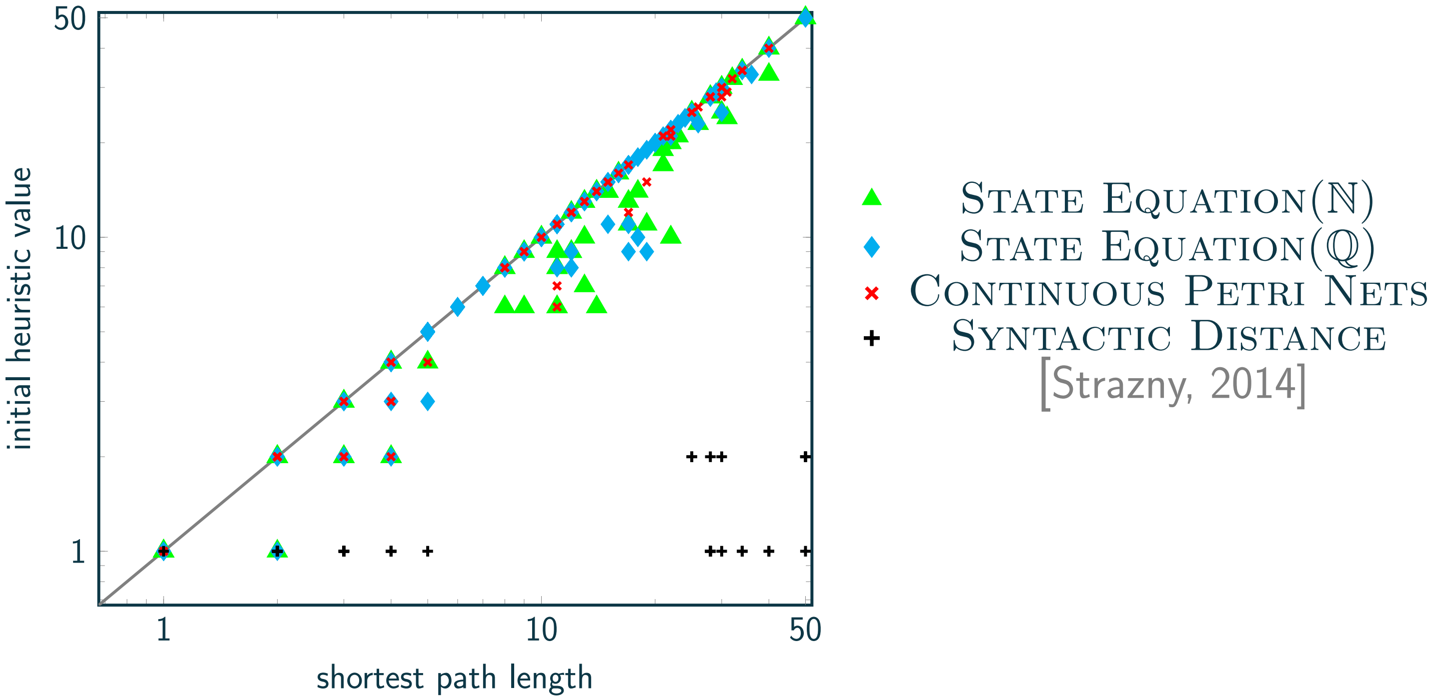

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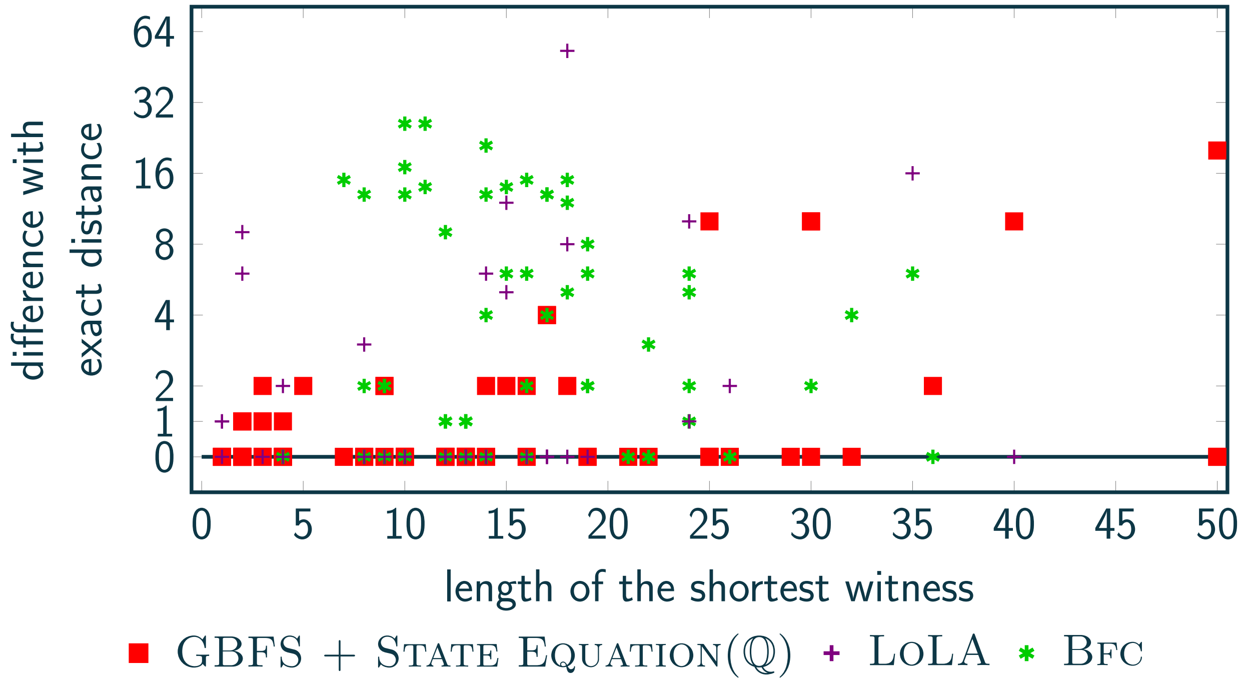

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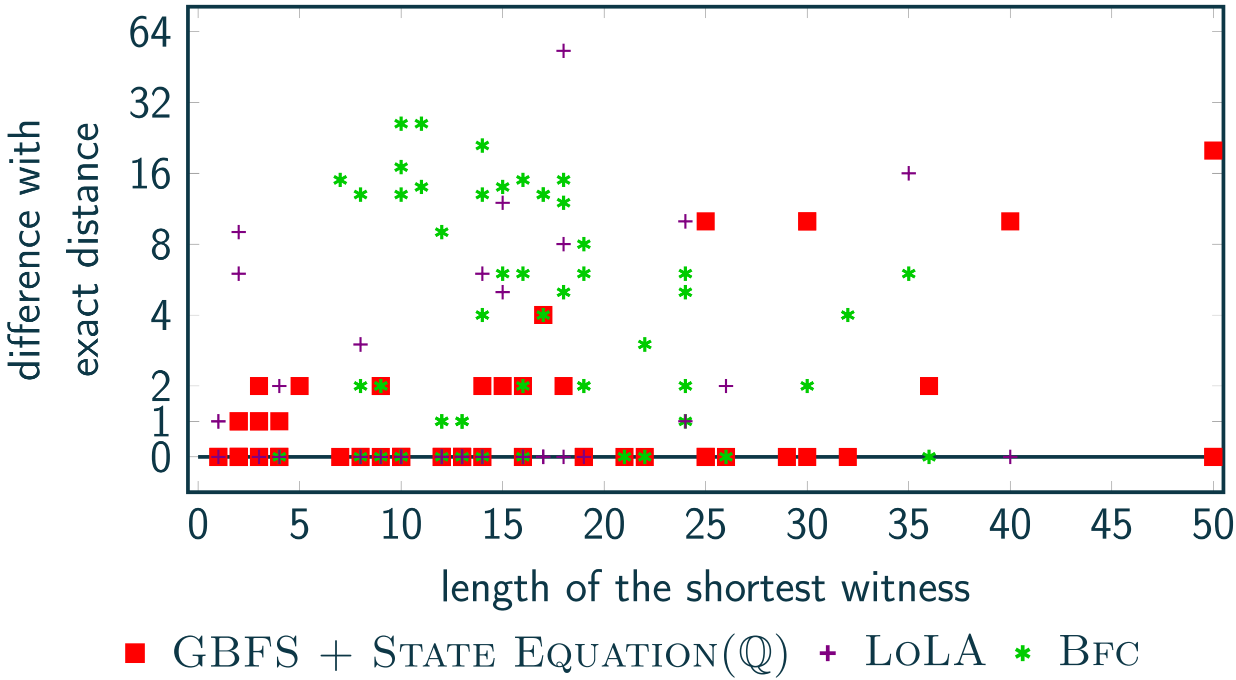

- A* is competitive against Bfc and much faster than LoLA while guaranteeing a shortest path

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- Directed search using these approximations is efficient
  ... even against domain specific solvers for coverability

Outlook

- Extension to directed model checking:
  For example, finding cycles with conditions (for LTL, ...)
- Representing overapproximations concisely
- Classes of Petri Nets with guarantees on heuristic accuracy
- Applying directed search to (undecidable) extensions
  (Transfer nets, reset nets, colored nets, ...)