The complexity of soundness in workflow nets

Philip Offtermatt

Joint work with
Michael Blondin and Filip Mazowiecki

MAX PLANCK INSTITUTE
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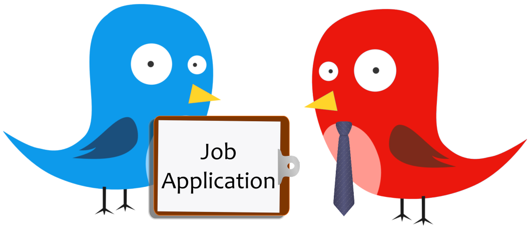

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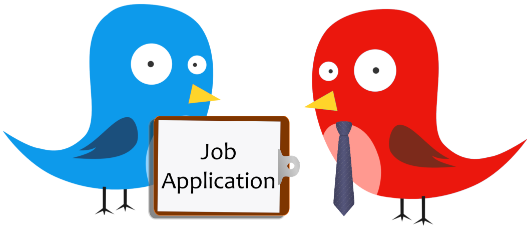

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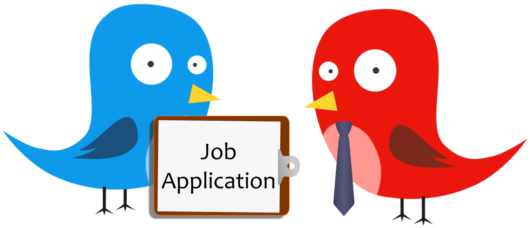

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How many applicants will we need until we find a new hire?

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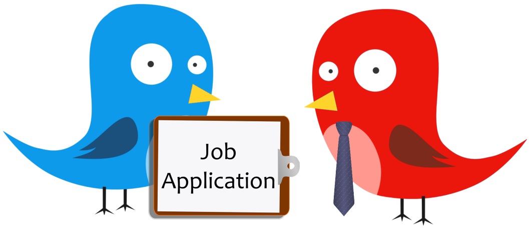

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Can we handle applications faster?

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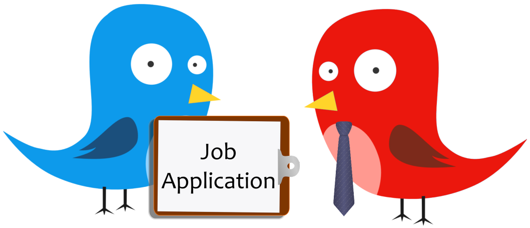

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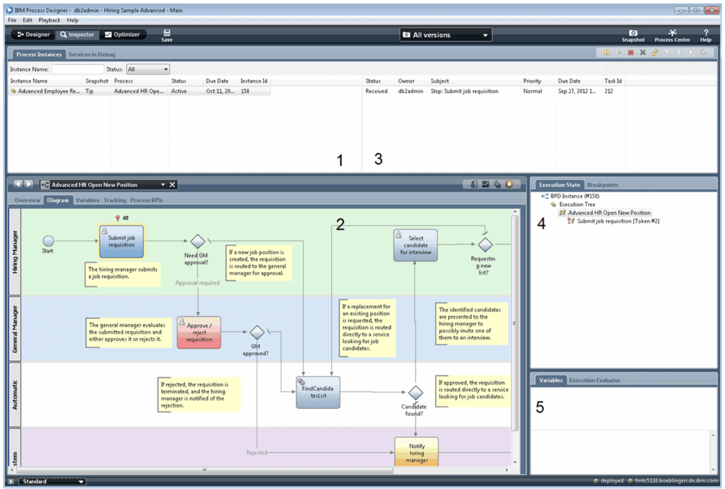
How many applicants will we need until we find a new hire?

Can we handle applications faster?

Will every applicant hear back from us?

Processes are everywhere!

Modelled by humans...



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Modelled by humans...

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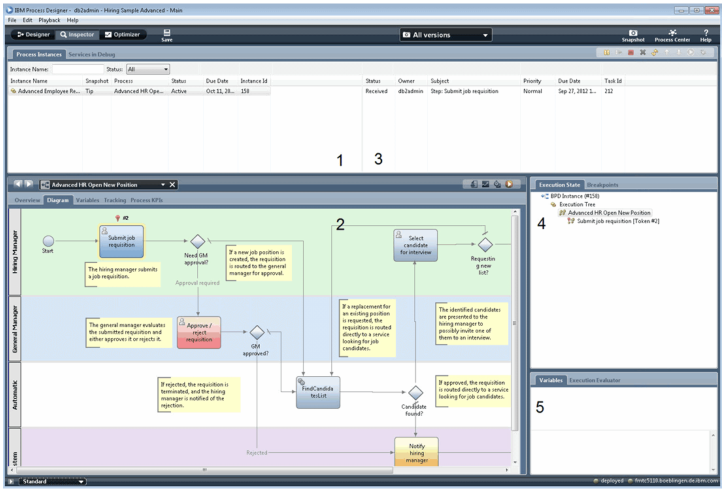

Case ID	Task Name	Resource	Date	Time
1	Receive Application	Peter	04/12/2020	06:37:11
1	Check legal requirements	Anne	05/12/2020	19:21:54
2	Receive Application	Peter	05/12/2020	02:04:19
3	Receive Application	Peter	06/12/2020	11:27:20
1	Check applicant suitability	Eva	06/12/2020	11:25:53
4	Receive Application	Peter	06/12/2020	14:18:20
5	Receive Application	Peter	07/12/2020	12:54:57
2	Check applicant suitability	Eva	08/12/2020	17:20:30
1	Accept	Eva	08/12/2020	06:45:23
3	Check legal requirements	Anne	08/12/2020	06:36:26
4	Check applicant suitability	Eva	16/12/2020	00:21:57
2	Check legal requirements	Anne	16/12/2020	09:03:05
2	Recheck Application	Chris	18/12/2020	19:44:24
2	Check legal requirements	Anne	19/12/2020	20:26:55
4	Check legal requirements	Anne	19/12/2020	17:38:49
4	Reject	Chris	20/12/2020	09:37:59
3	Check applicant suitability	Anne	20/12/2020	01:32:44
2	Check applicant suitability	Peter	27/12/2020	03:35:57
3	Accept	Eva	29/12/2020	17:18:55
2	Reject	Peter	29/12/2020	03:48:06
5	Check legal requirements	Anne	29/12/2020	03:37:39

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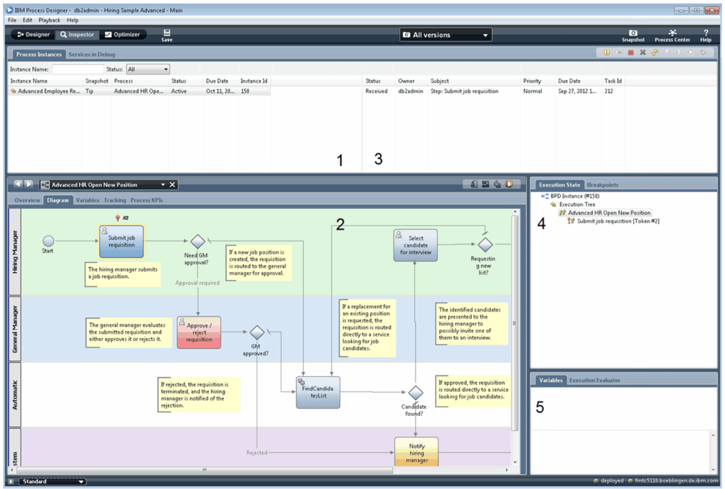

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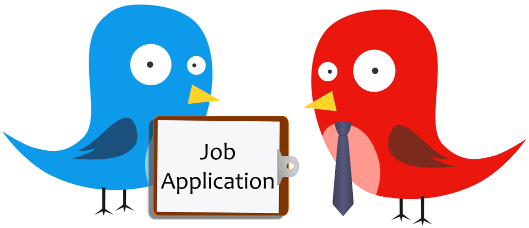
How can we formally reason about processes?

Formally modelling processes: Workflow nets



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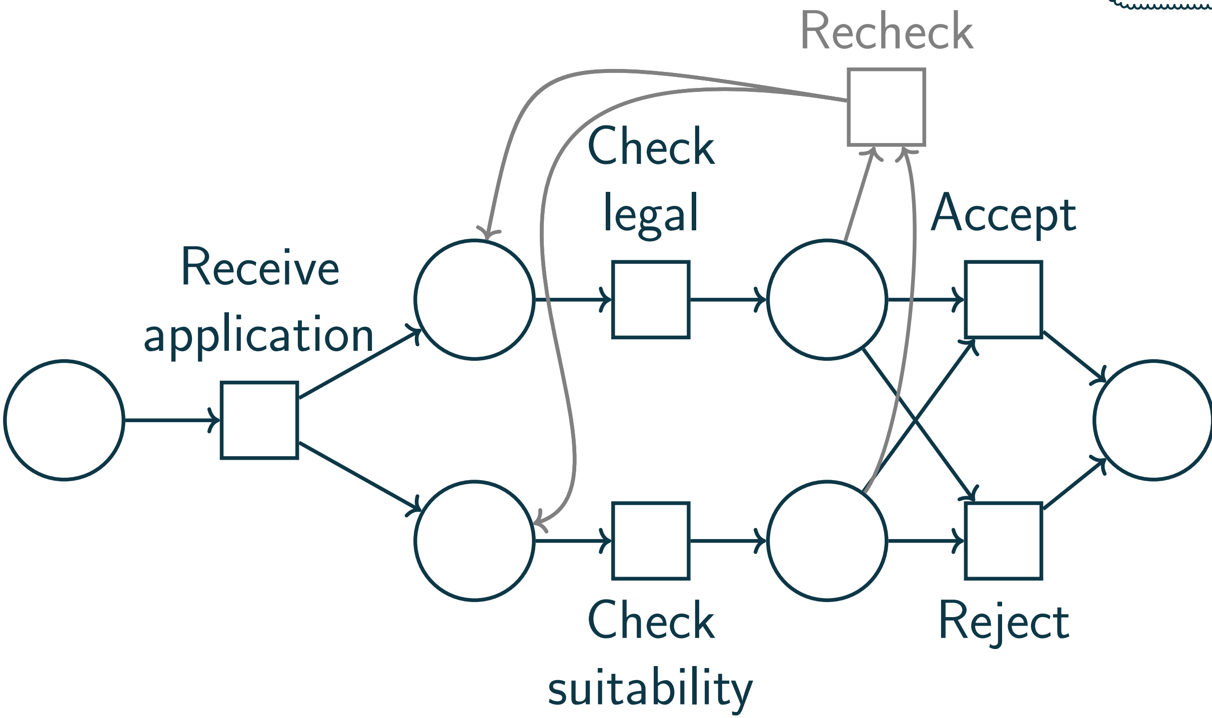

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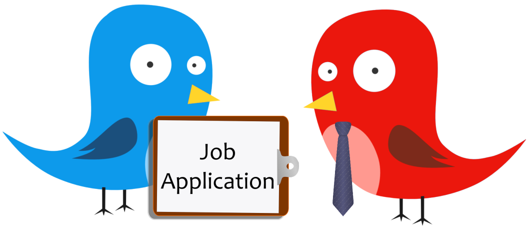

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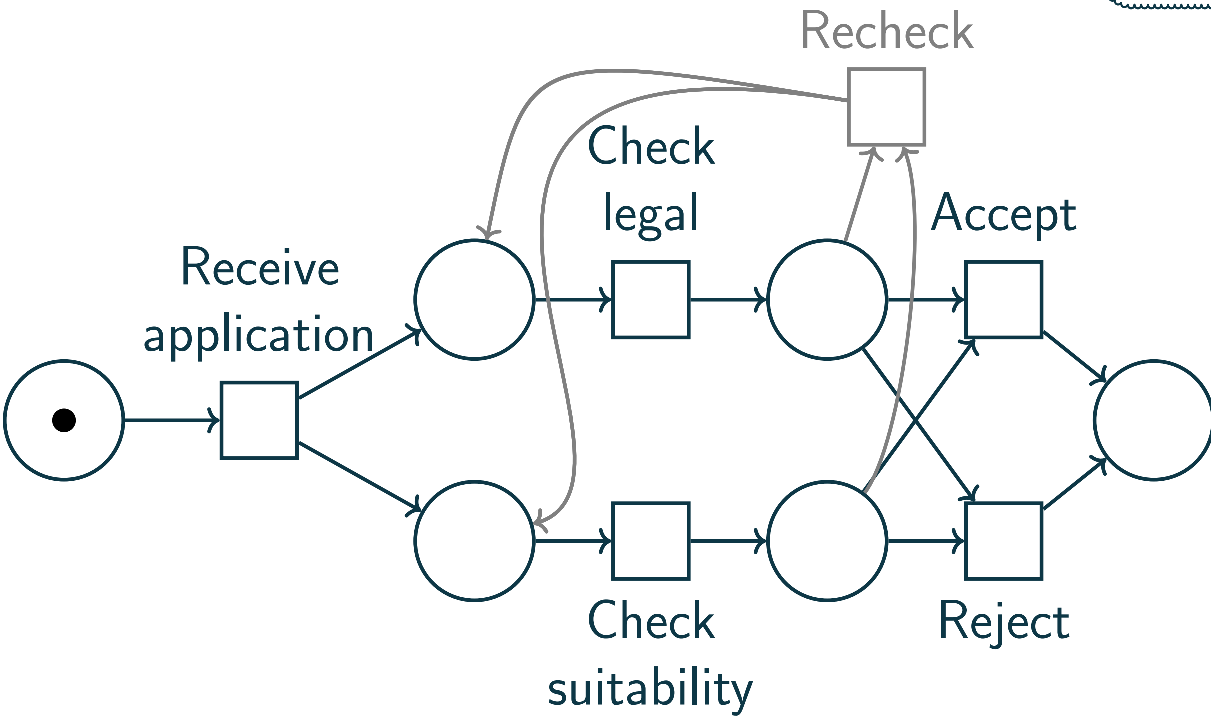

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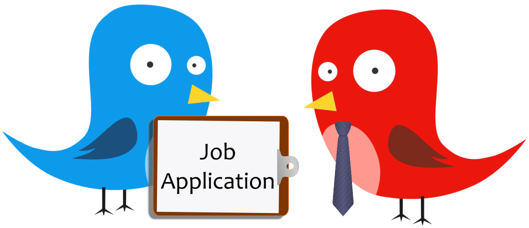

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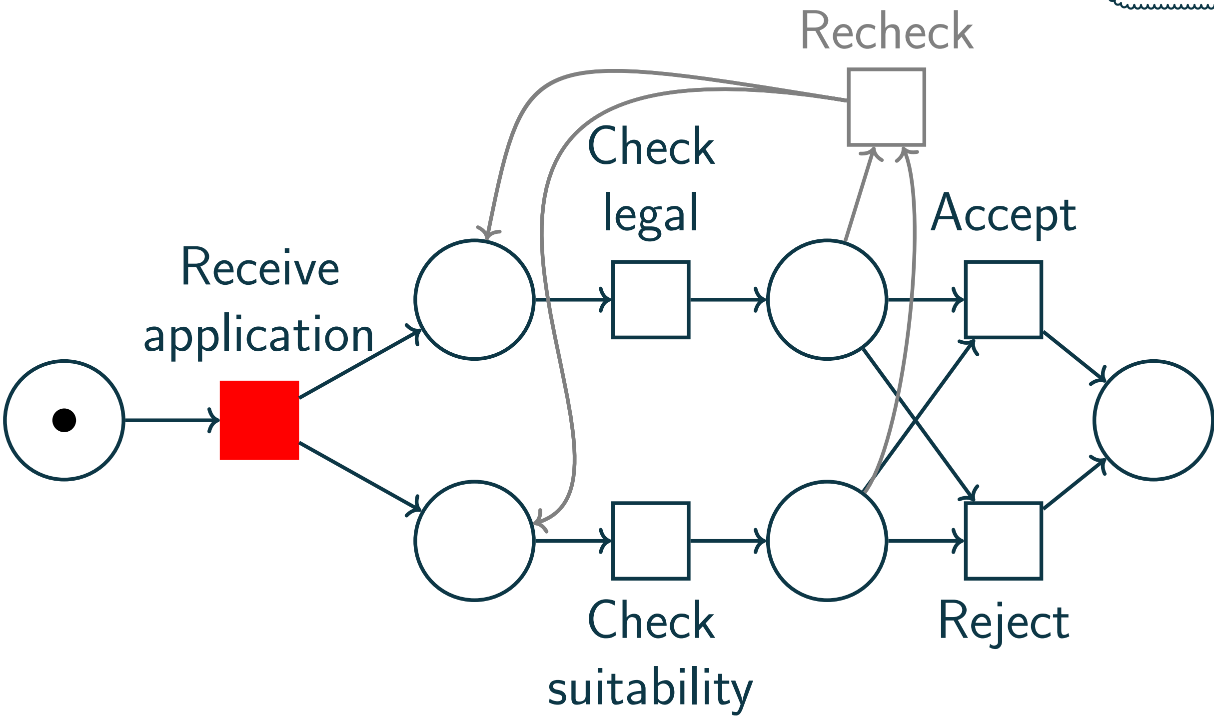

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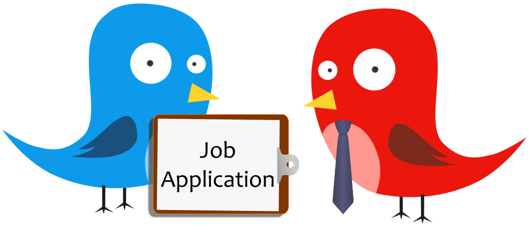

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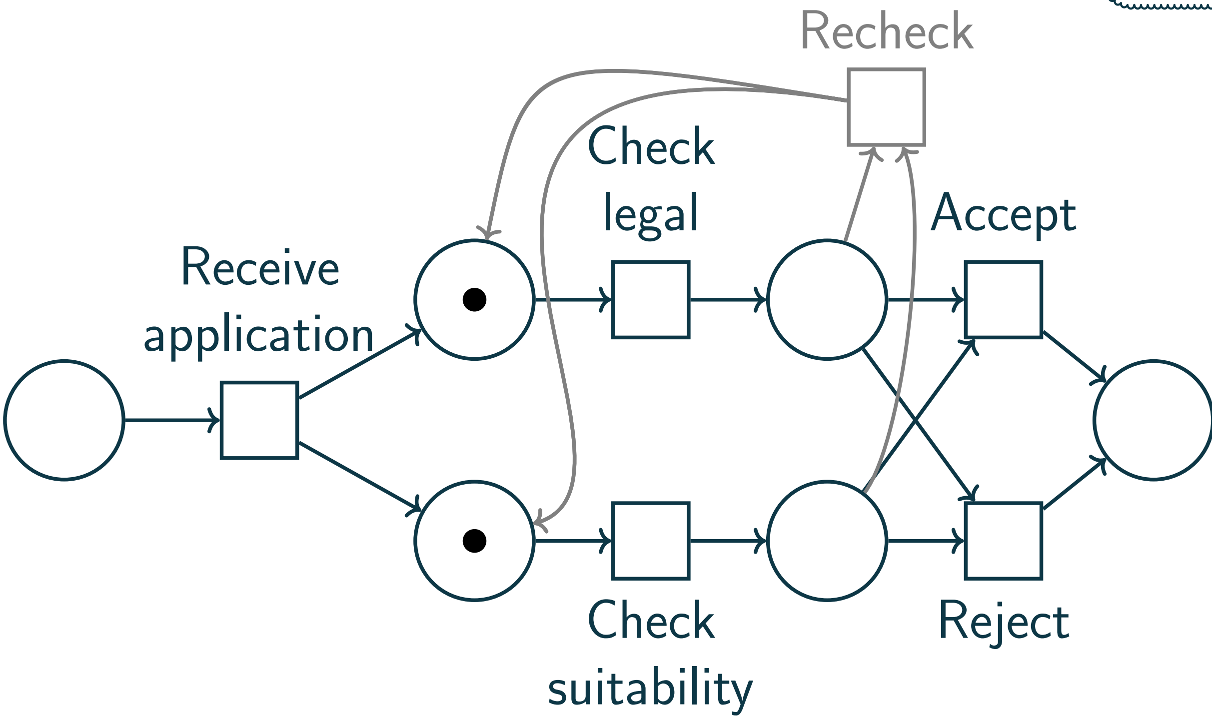

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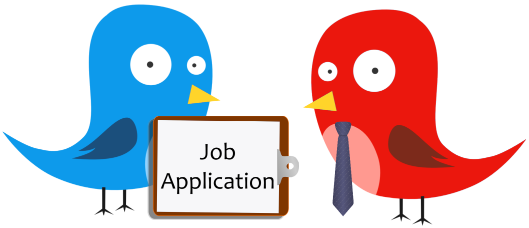

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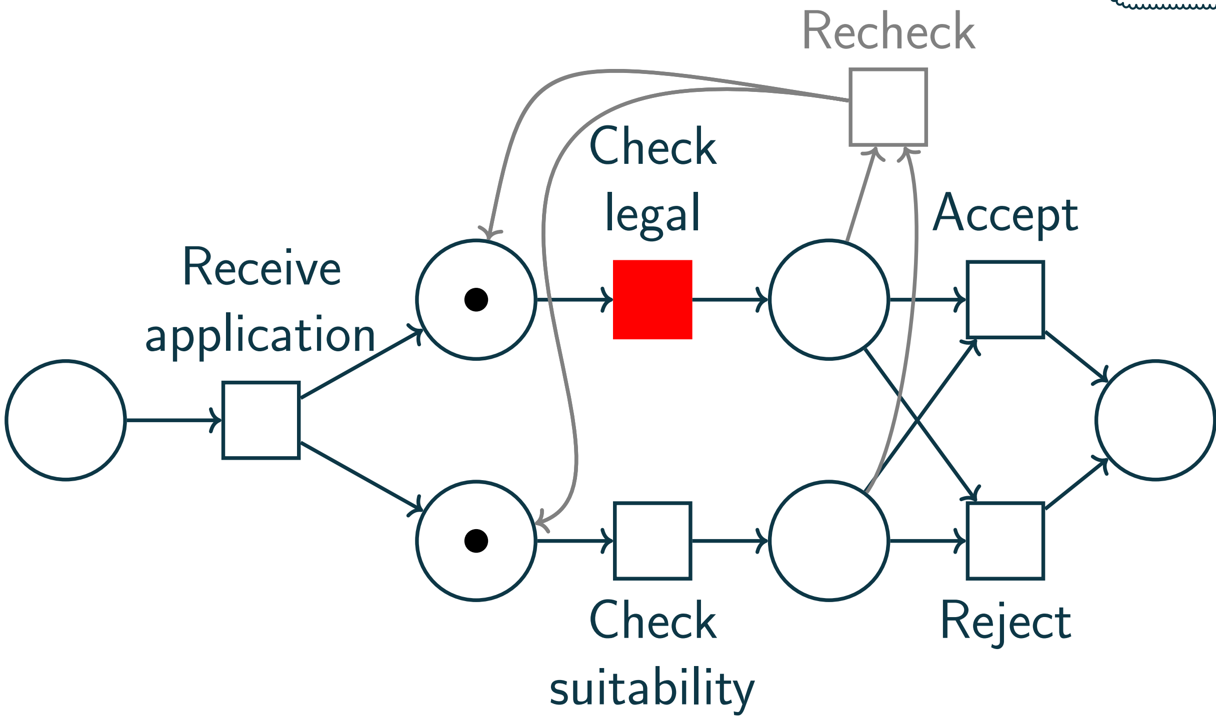

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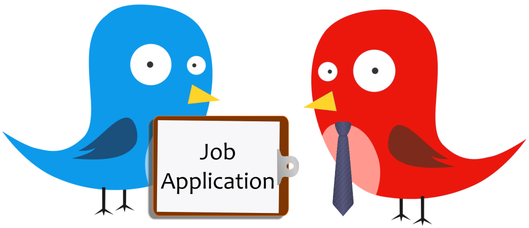

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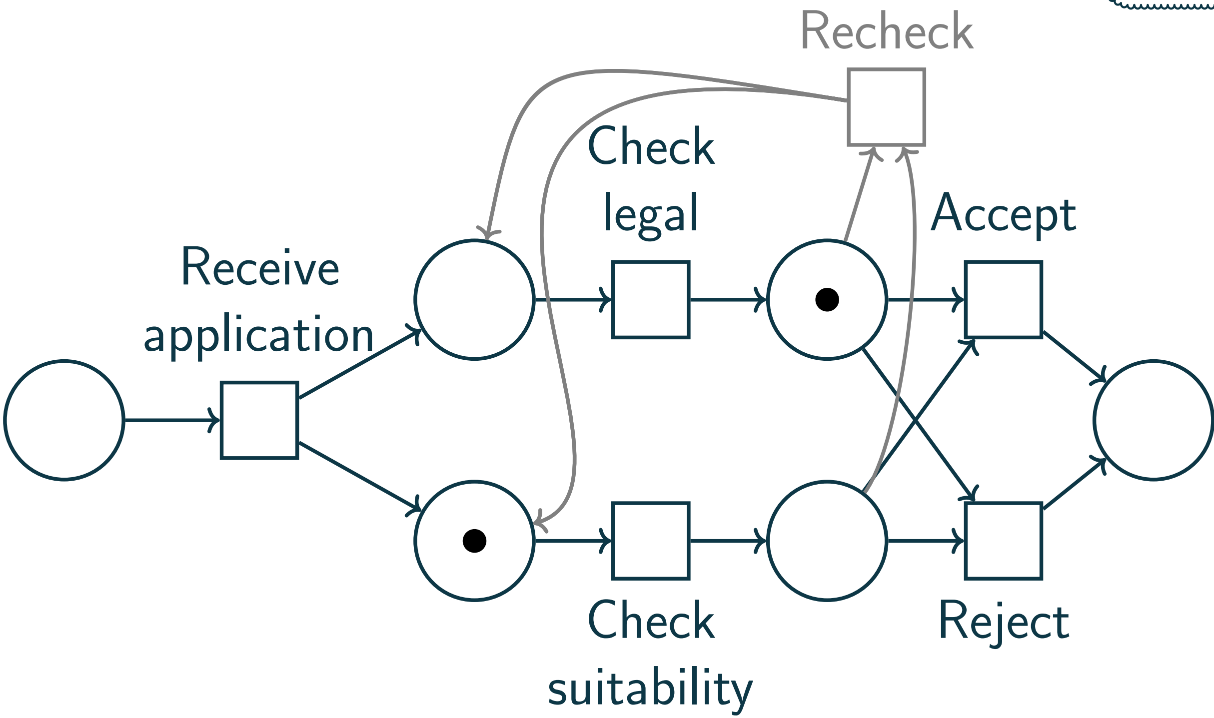

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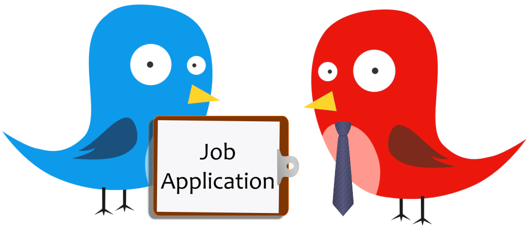

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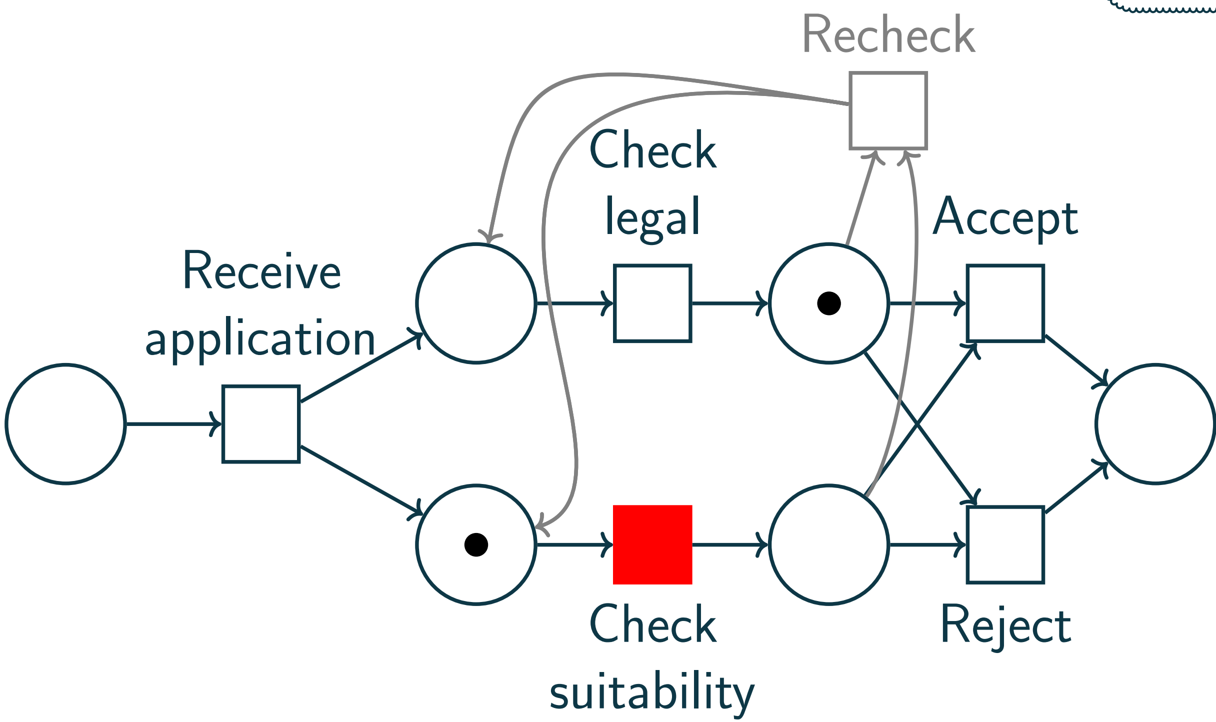

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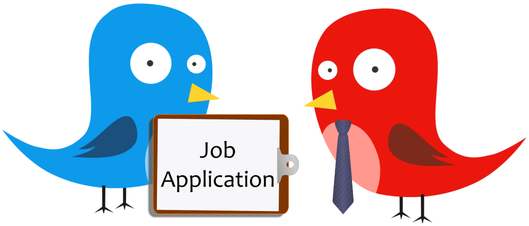

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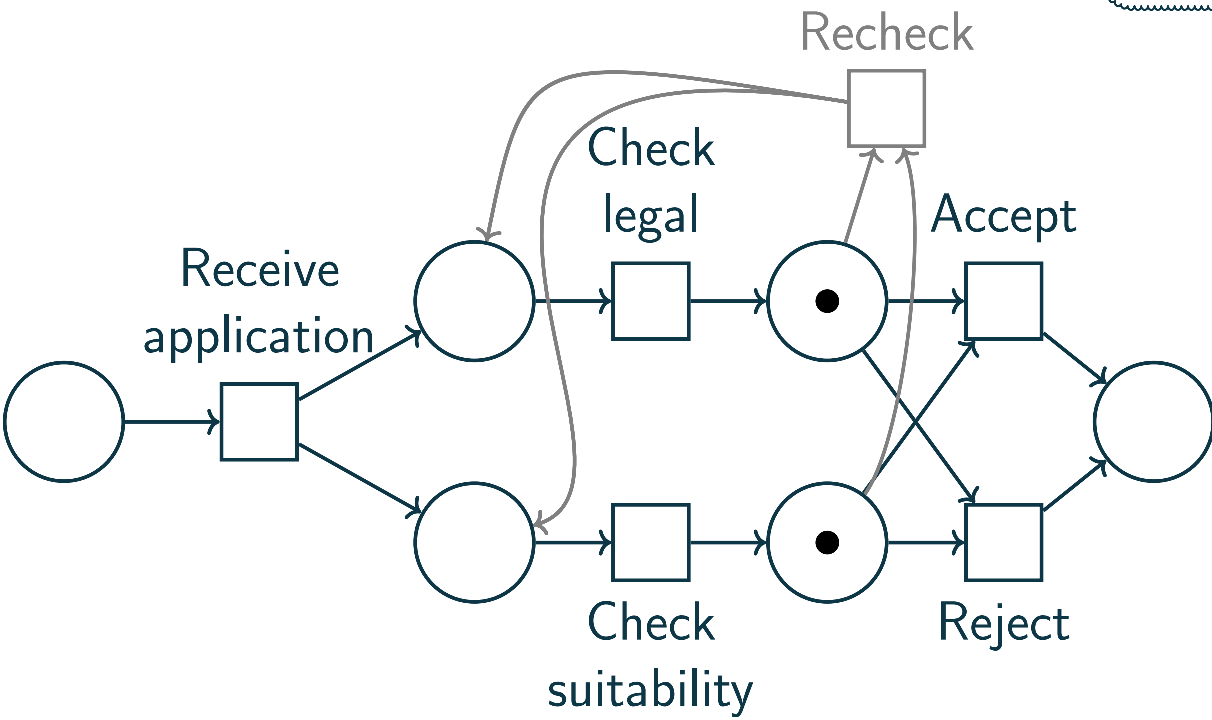

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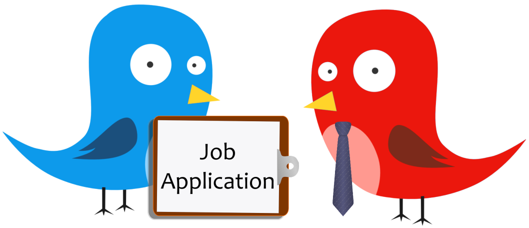

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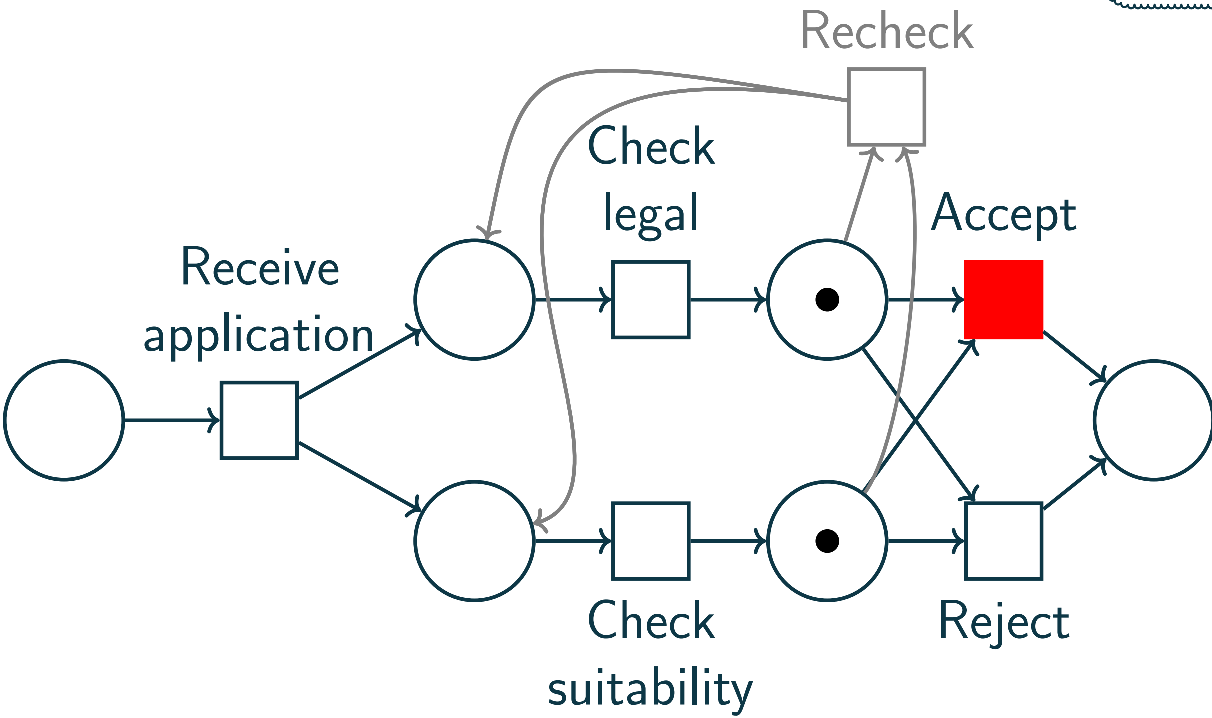

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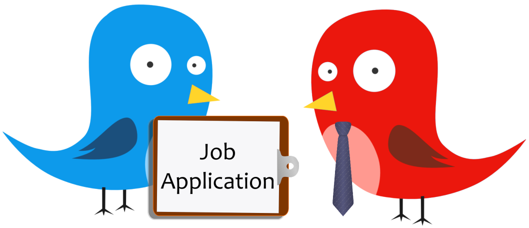

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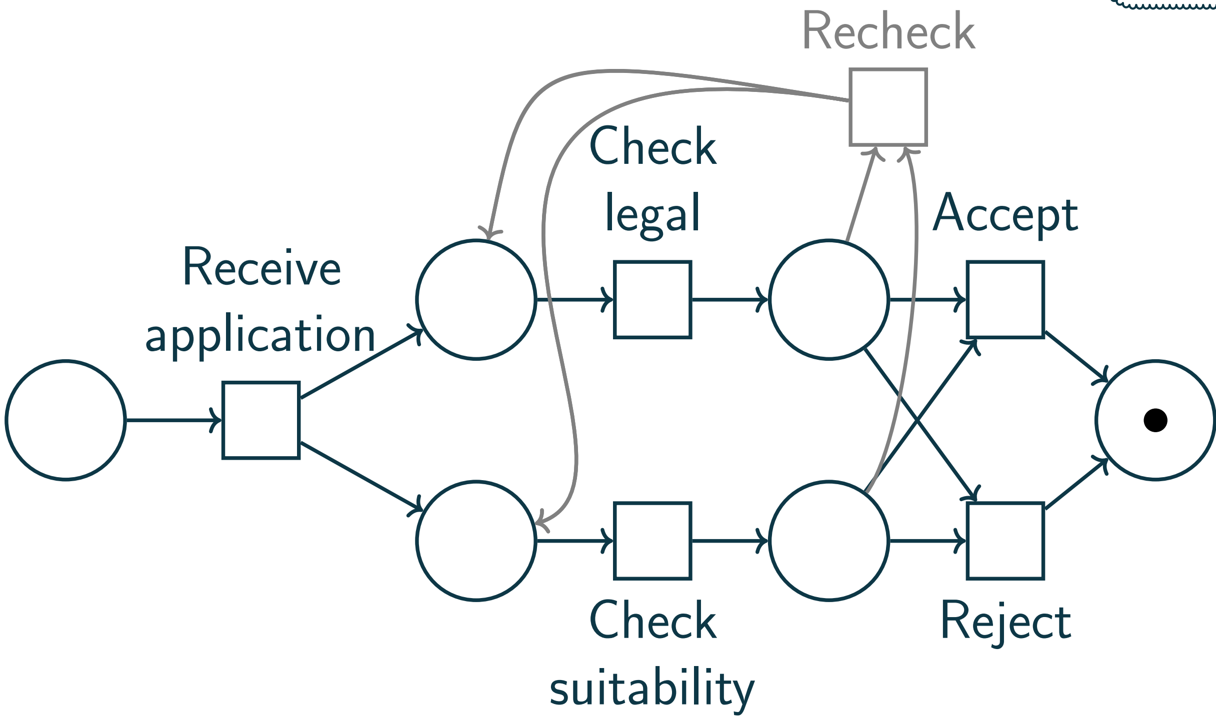
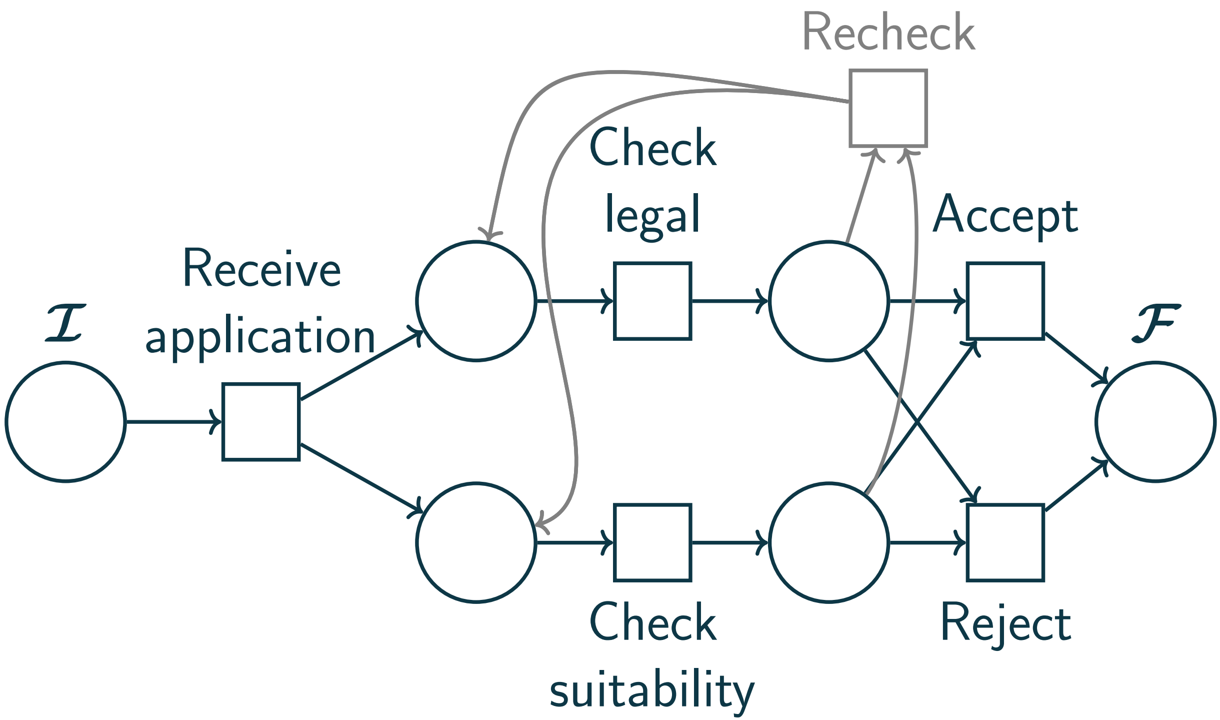


Workflow nets

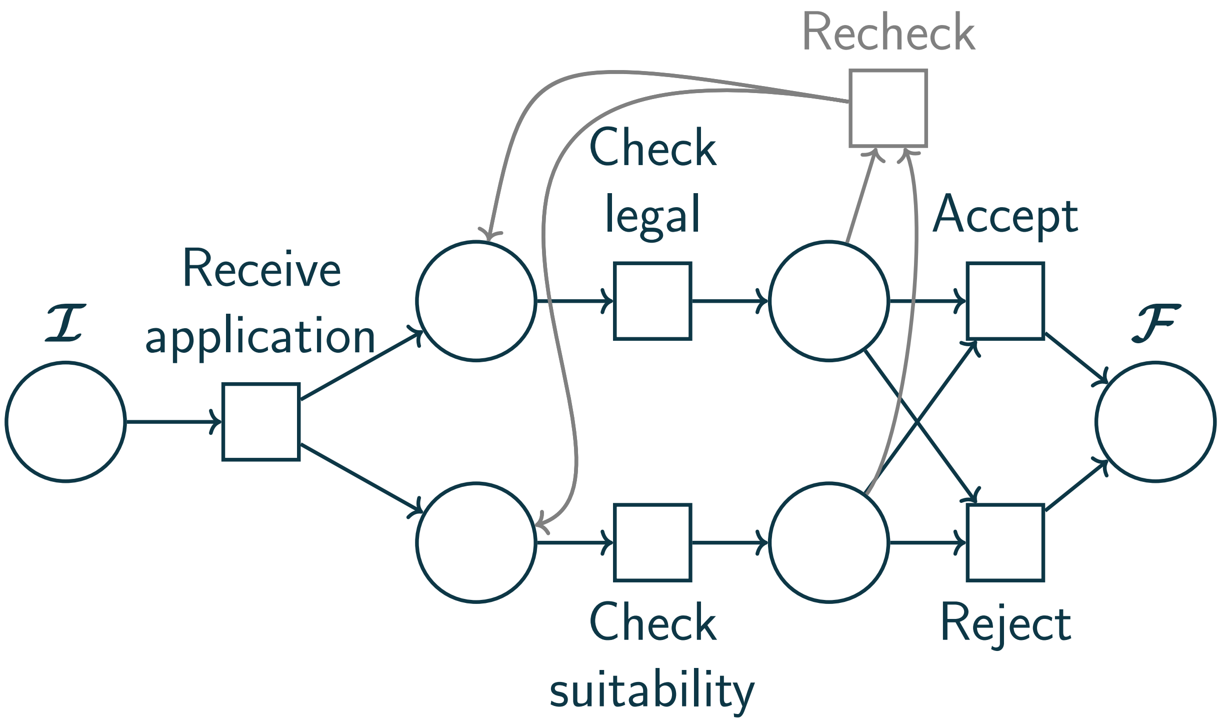
Formally: Petri nets of a specific shape



Workflow nets

Formally: Petri nets of a specific shape

1. $\mathcal{I}$ has no incoming arcs



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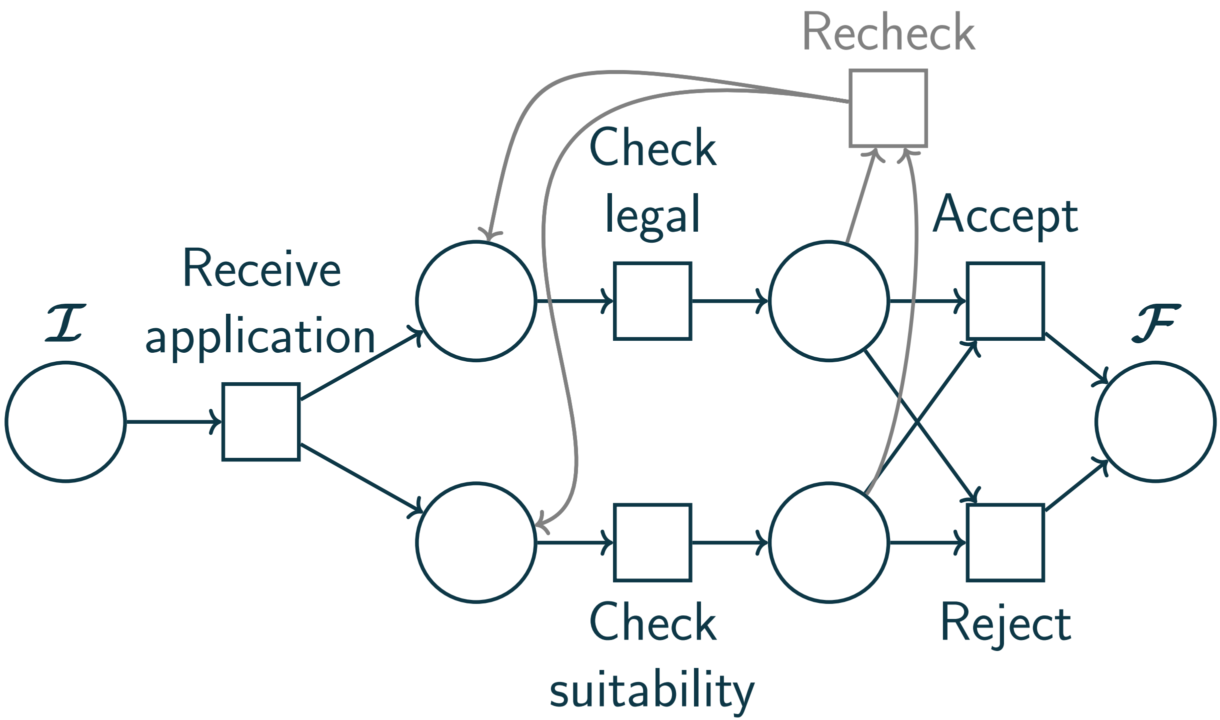

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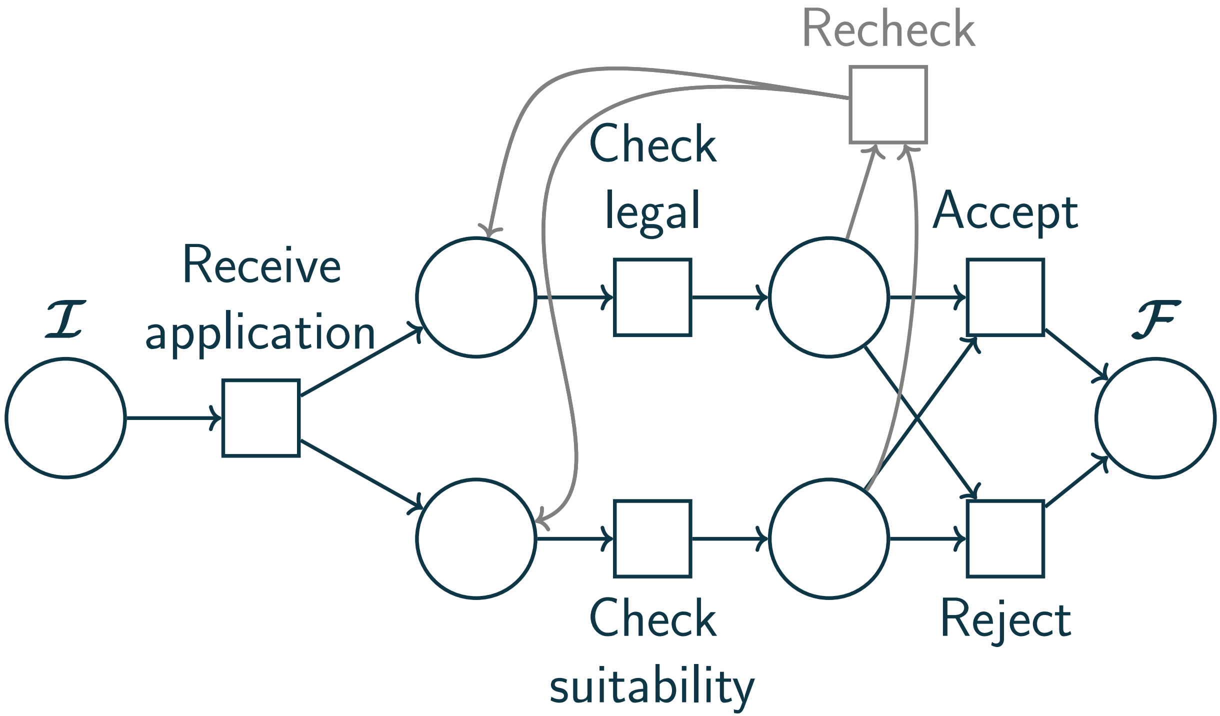
3. All transitions are on a path from $\mathcal{I}$ to $\mathcal{F}$



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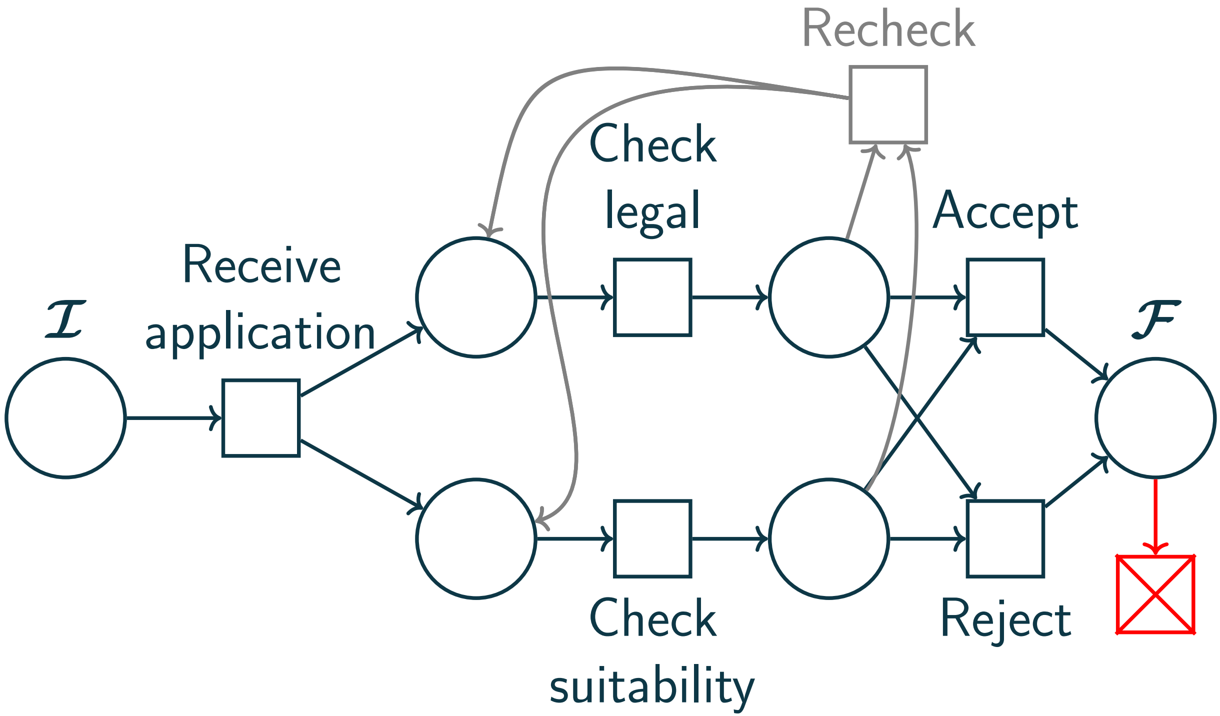
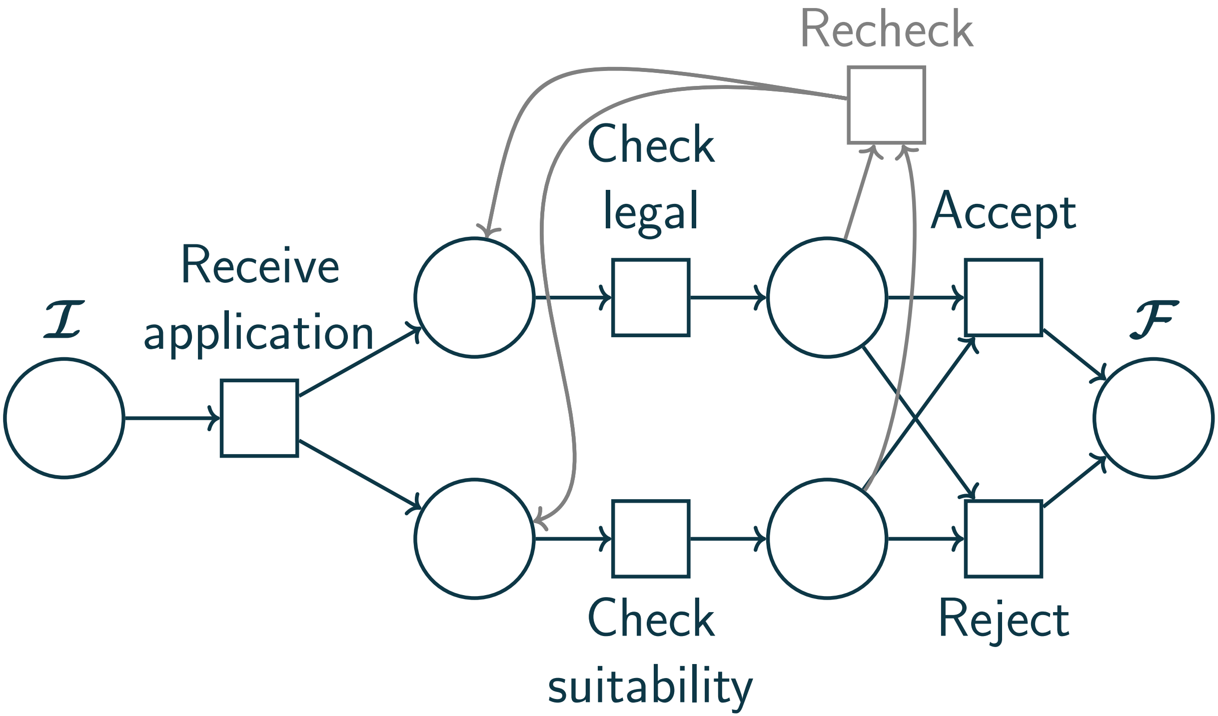


Correctness conditions for processes



Correctness conditions for processes

Option to complete:
We should be able to reach a
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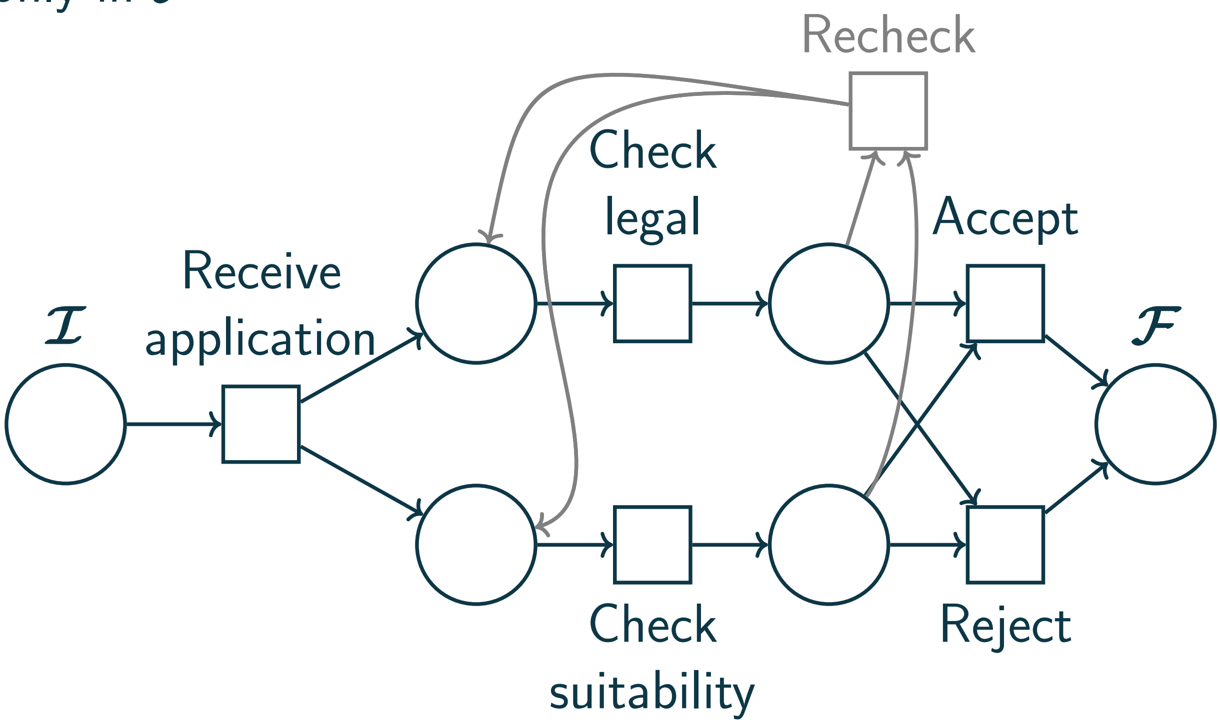
only in $\mathcal{F}$



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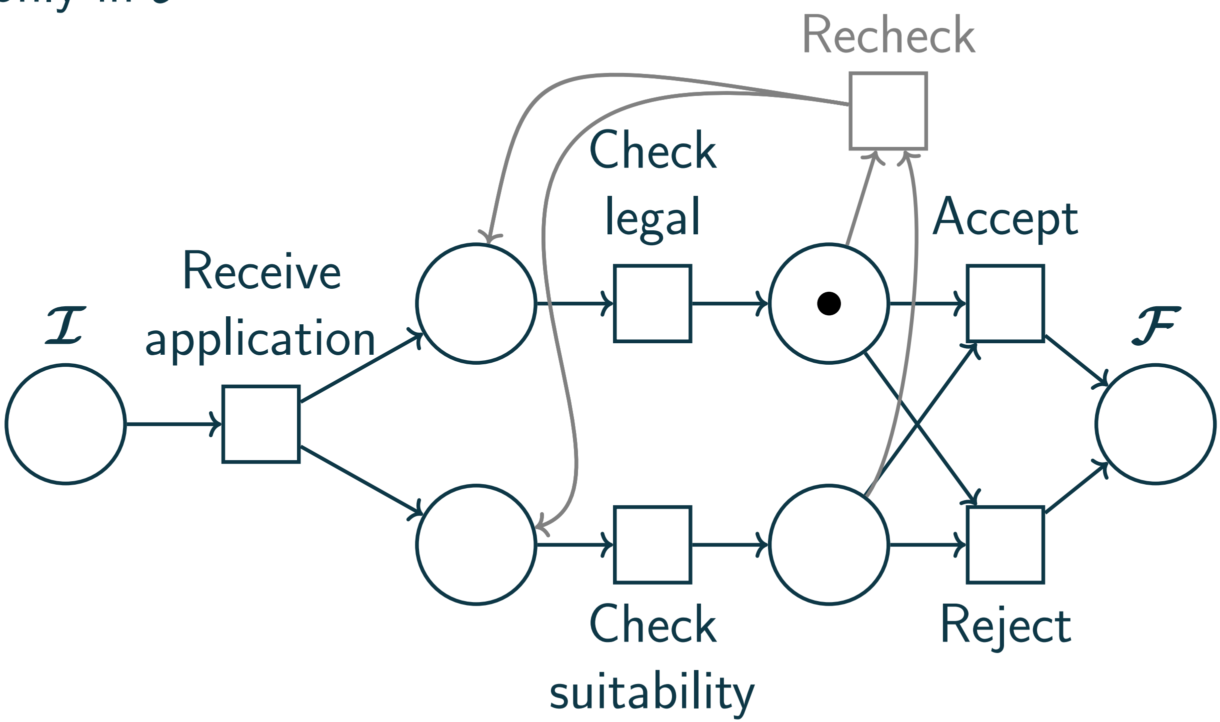

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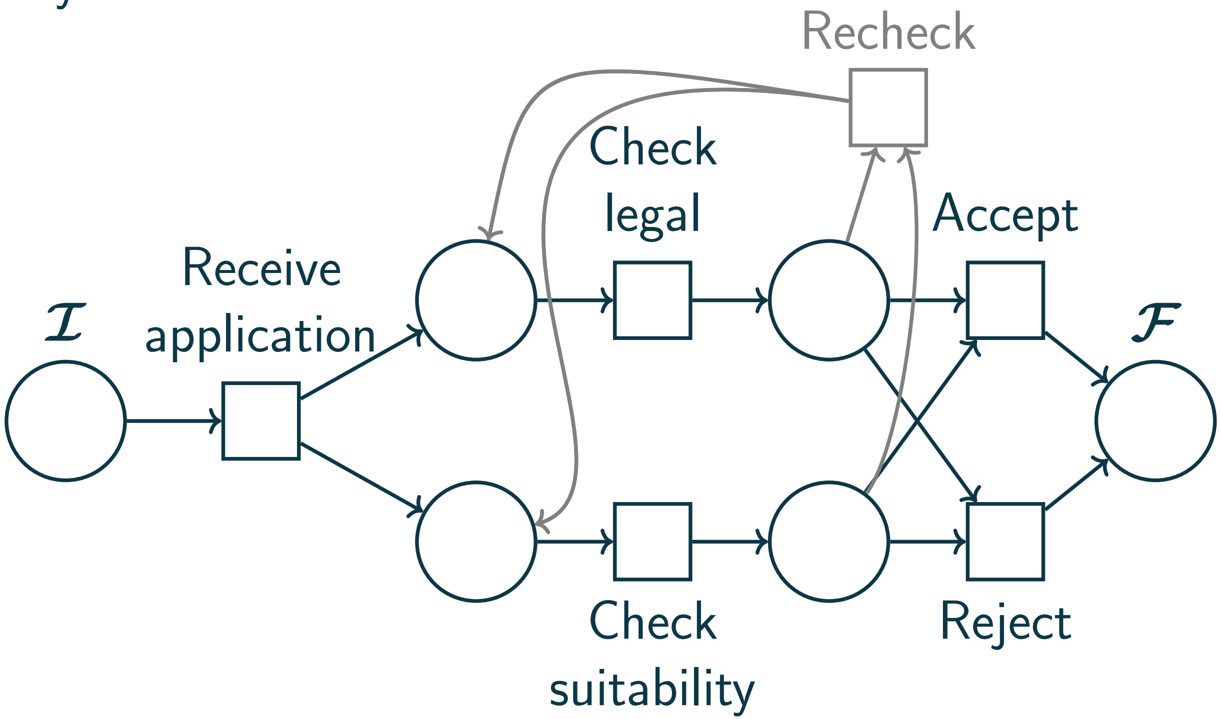
Proper completion:
When $\mathcal{F}$ is marked the rest of the net is empty



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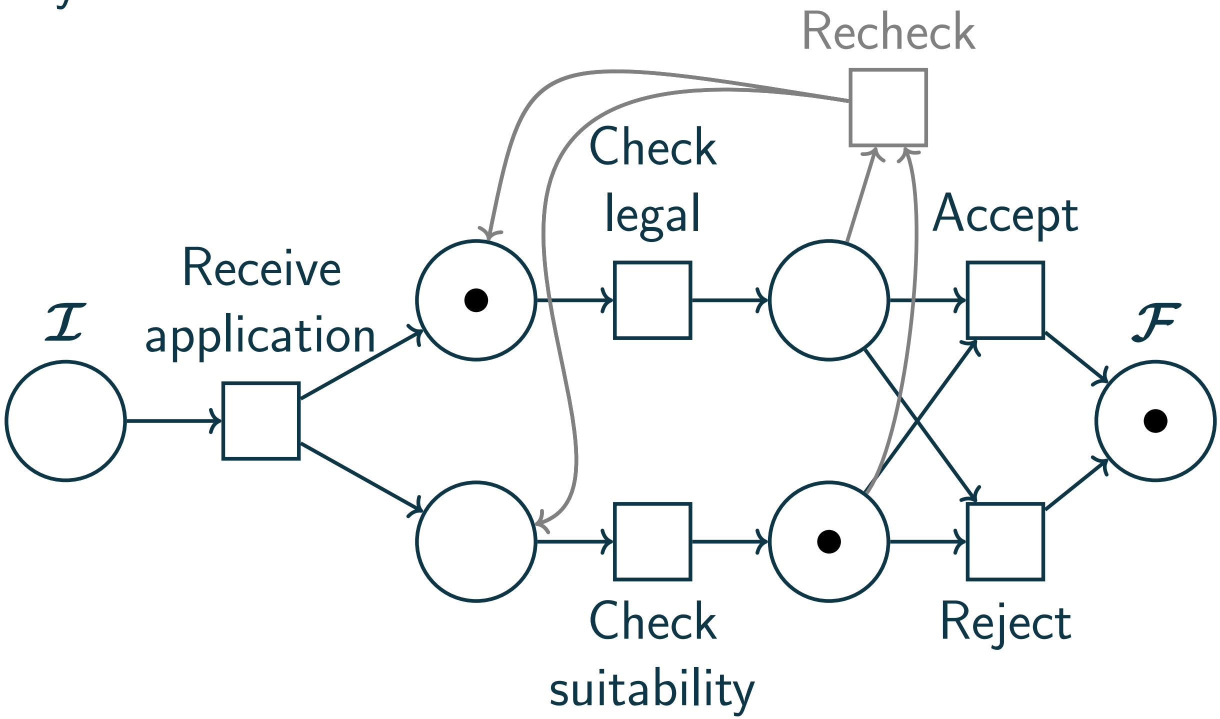

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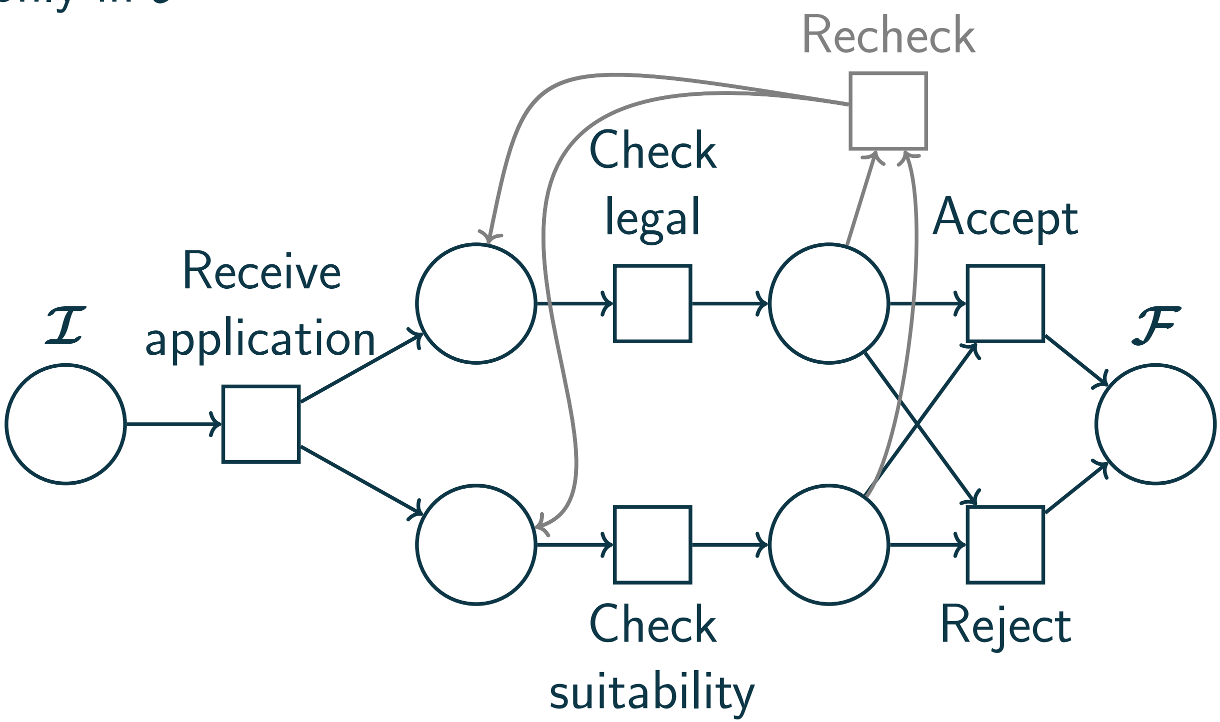
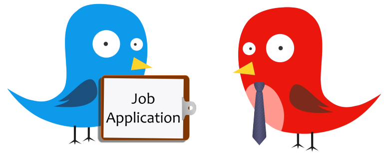


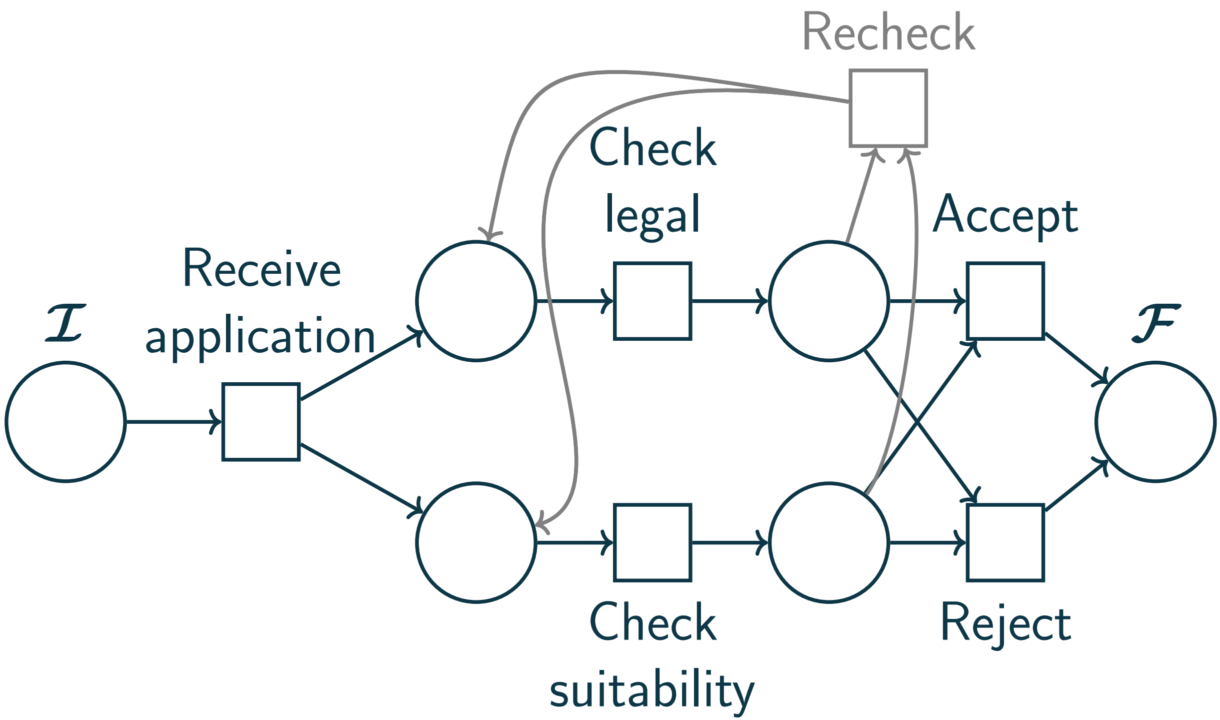
Can we condense these into a single condition?

A concise correctness condition

Soundness:
From any marking reachable from $\{\mathcal{I}: 1\}$, the final marking $\{\mathcal{F}: 1\}$ can be reached



$\forall$ runs π $\exists$ run π' : $\{\mathcal{I}: 1\} \xrightarrow{\pi\pi'} \{\mathcal{F}: 1\}$



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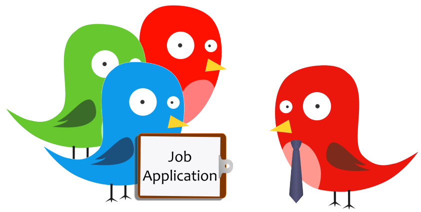
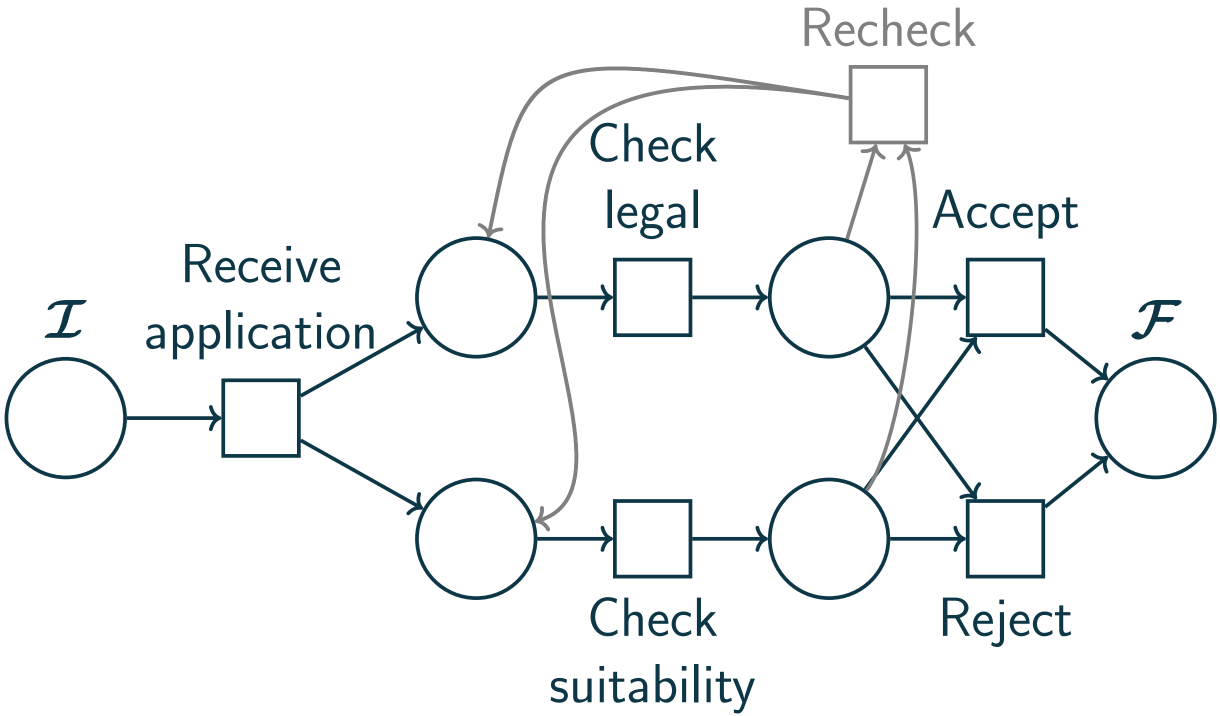
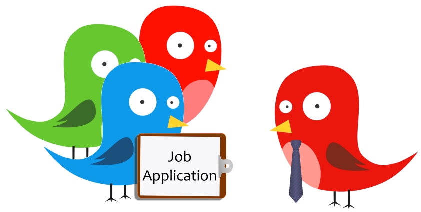
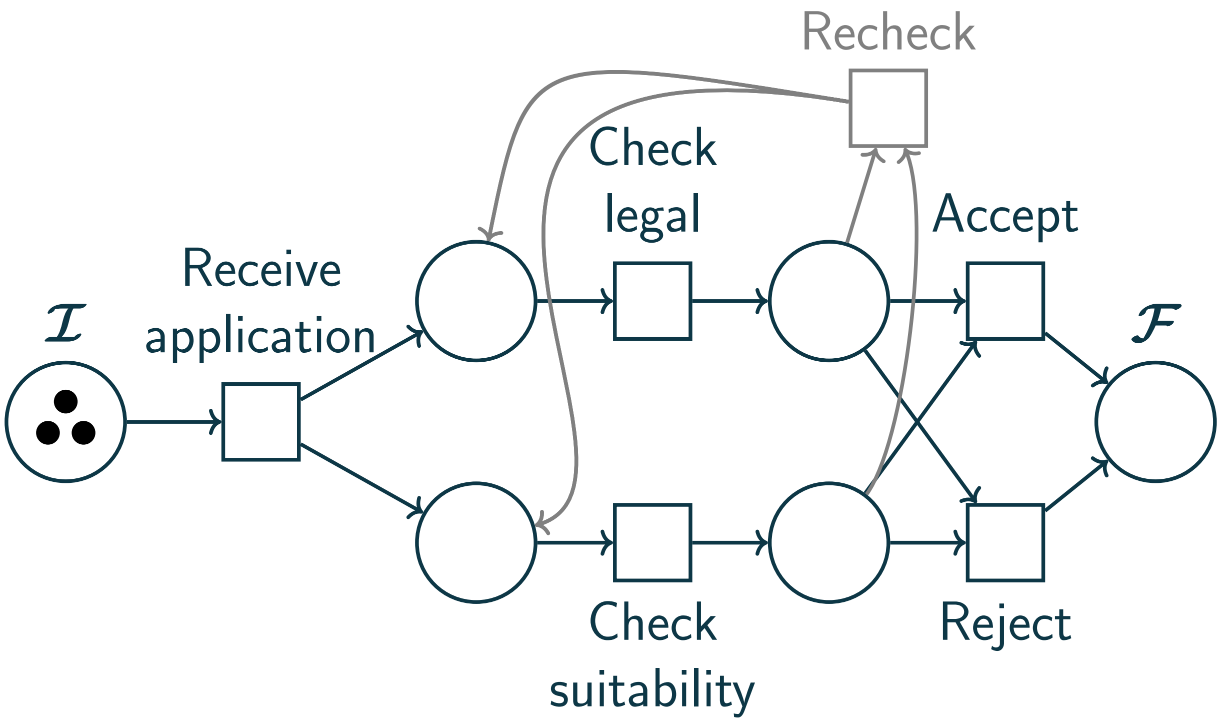




Extending soundness



k-**soundness:**
From any marking reachable from $\{\mathcal{I}: k\}$, the final marking $\{\mathcal{F}: k\}$ can be reached



Variants of soundness

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Structural soundness:
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Variants of soundness

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Generalised soundness:
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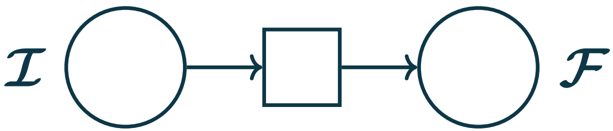

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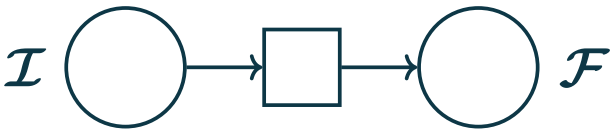	✓	

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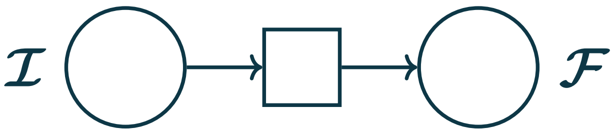	✓	✓

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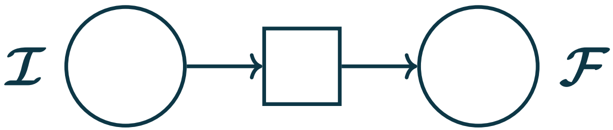
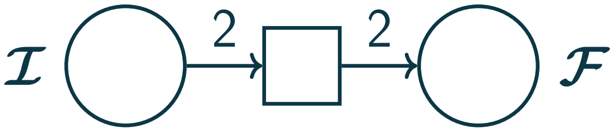
	Generalised soundness: $\forall k\colon$ k-sound	**Structural soundness:** $\exists k\colon$ k-sound
	✓	✓
		

Variants of soundness

k-**soundness:**
From any marking reachable from $\{\mathcal{I}\colon k\}$,
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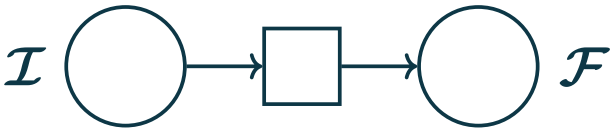
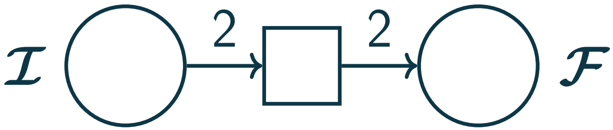
	✓	✓
	X Not 1-sound	

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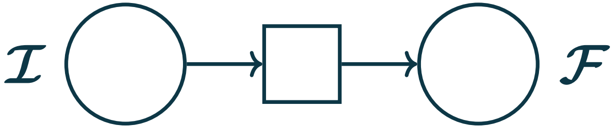
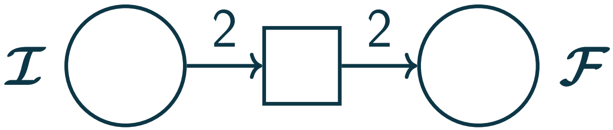
	✓	✓
	X Not 1-sound	✓ 2-sound

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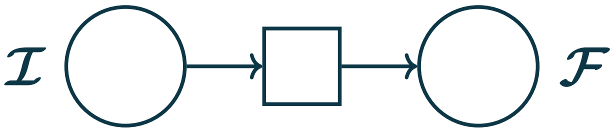
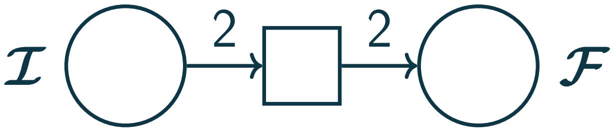
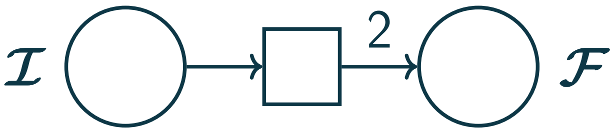
	✓	✓
	X Not 1-sound	✓ 2-sound
		

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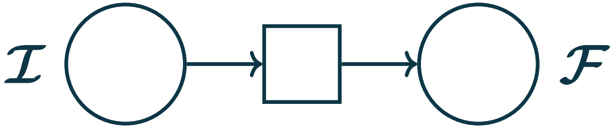
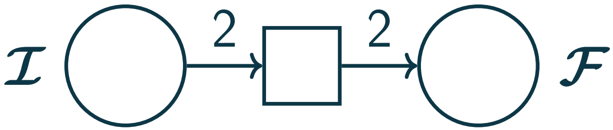
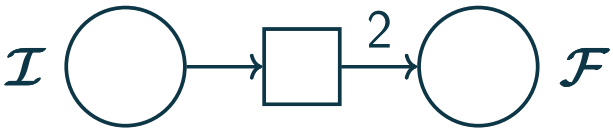
	✓	✓
	X Not 1-sound	✓ 2-sound
	X	

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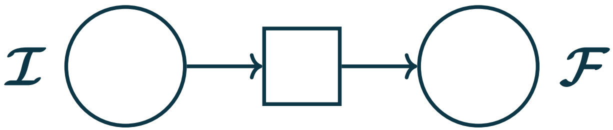
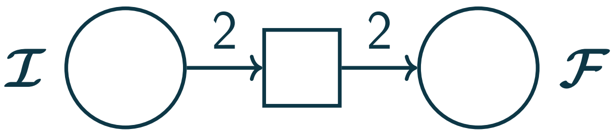
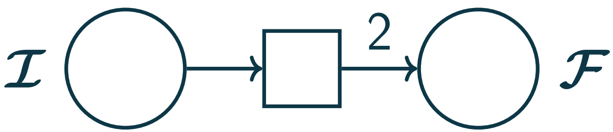
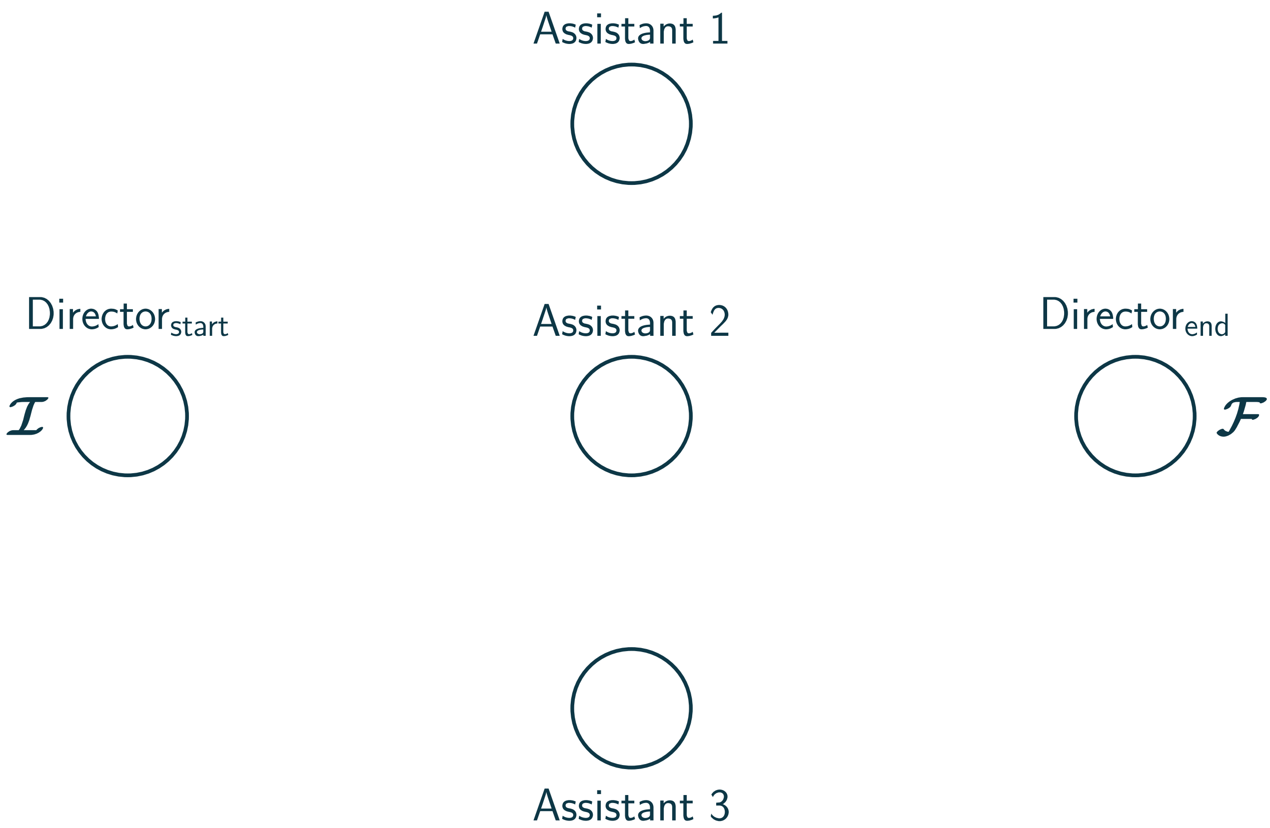
	✓	✓
	X Not 1-sound	✓ 2-sound
	X	X

Variants of soundness



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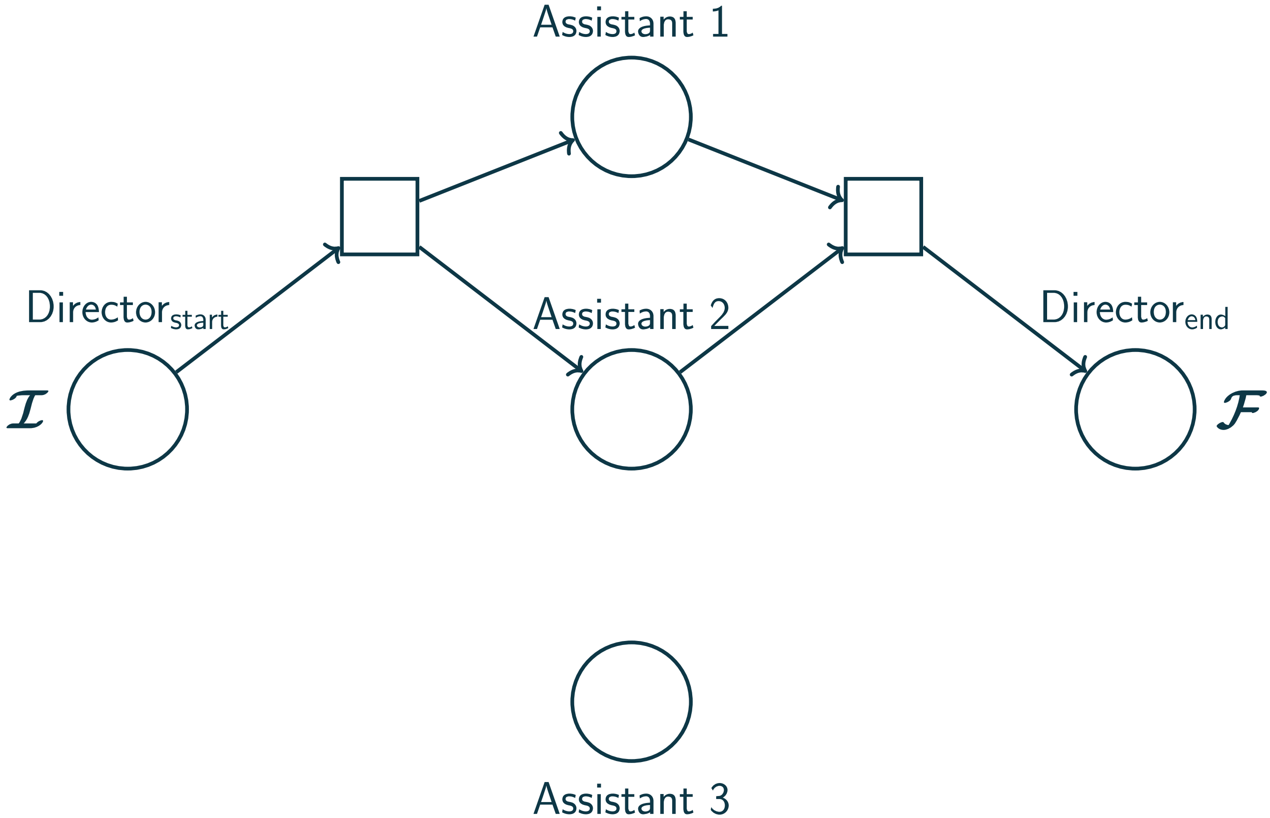

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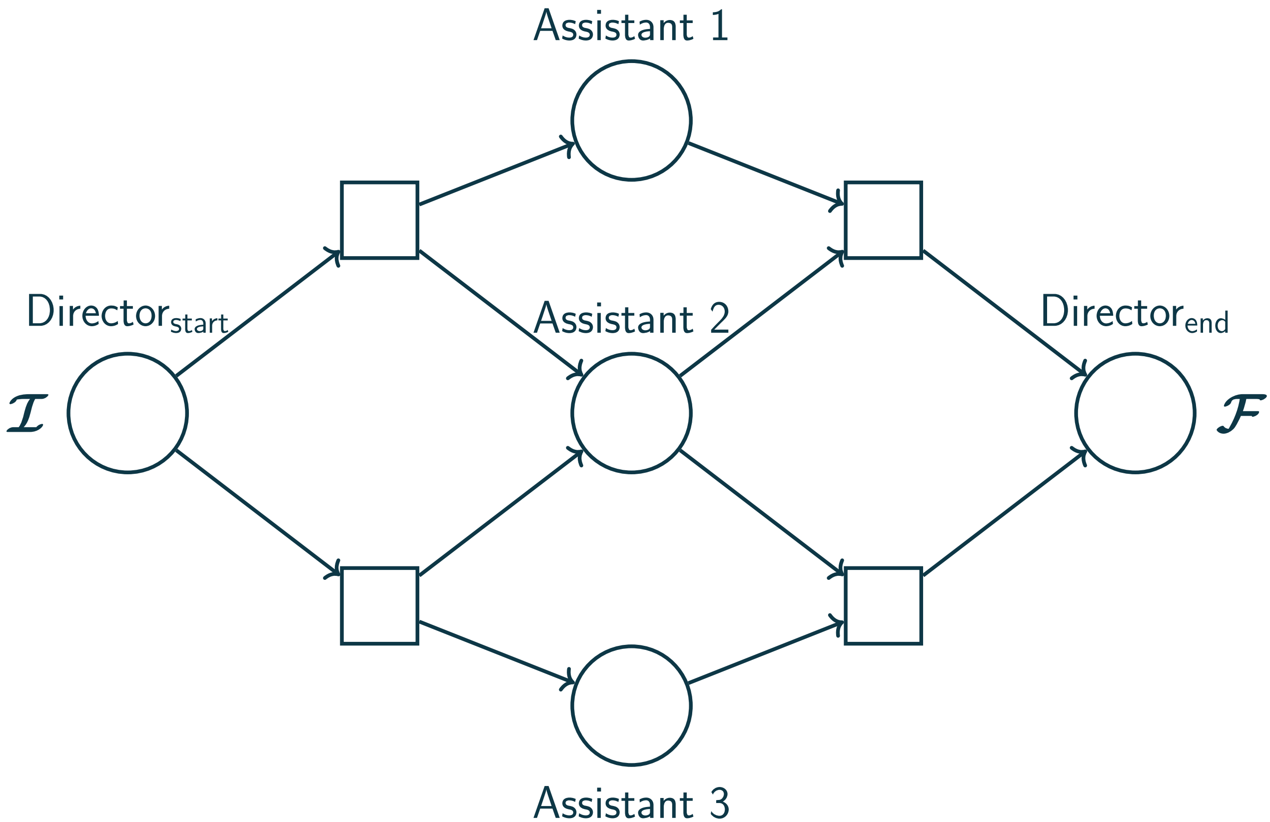

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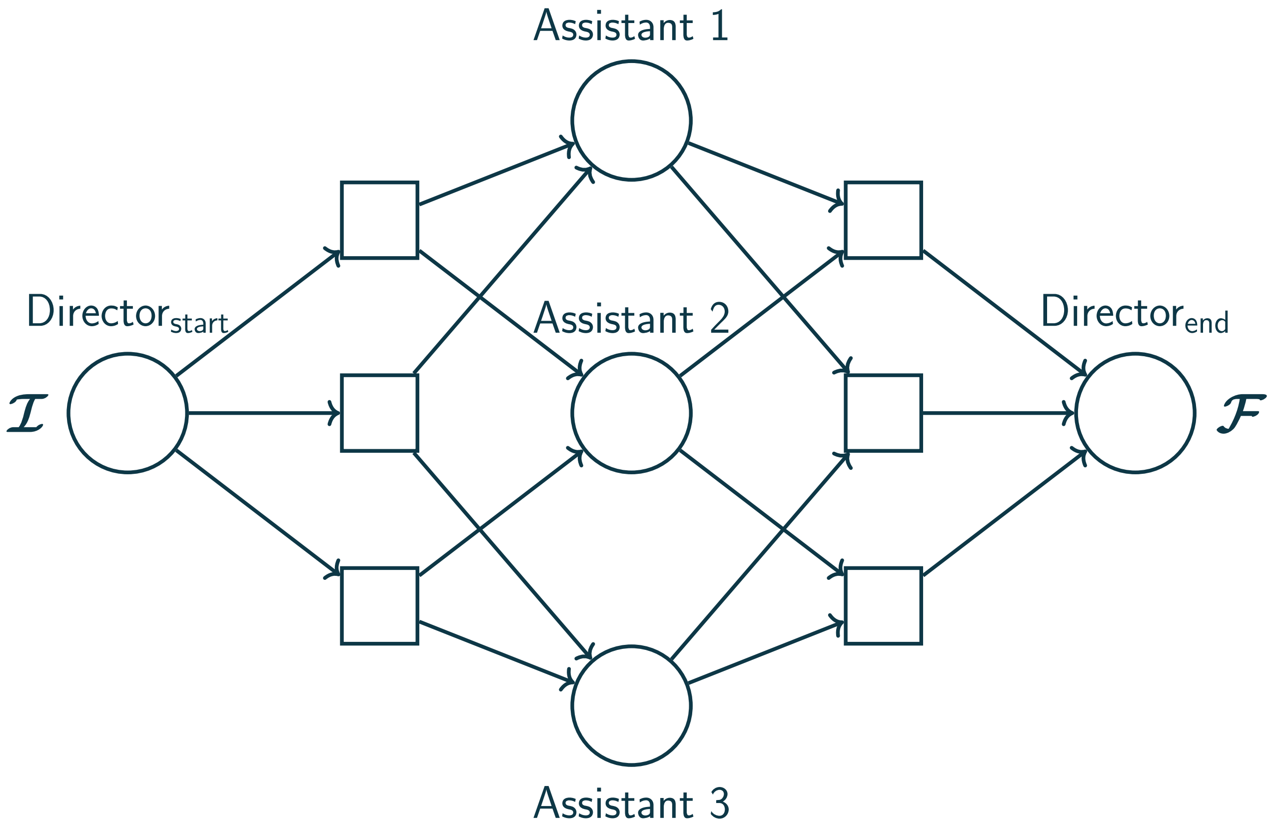

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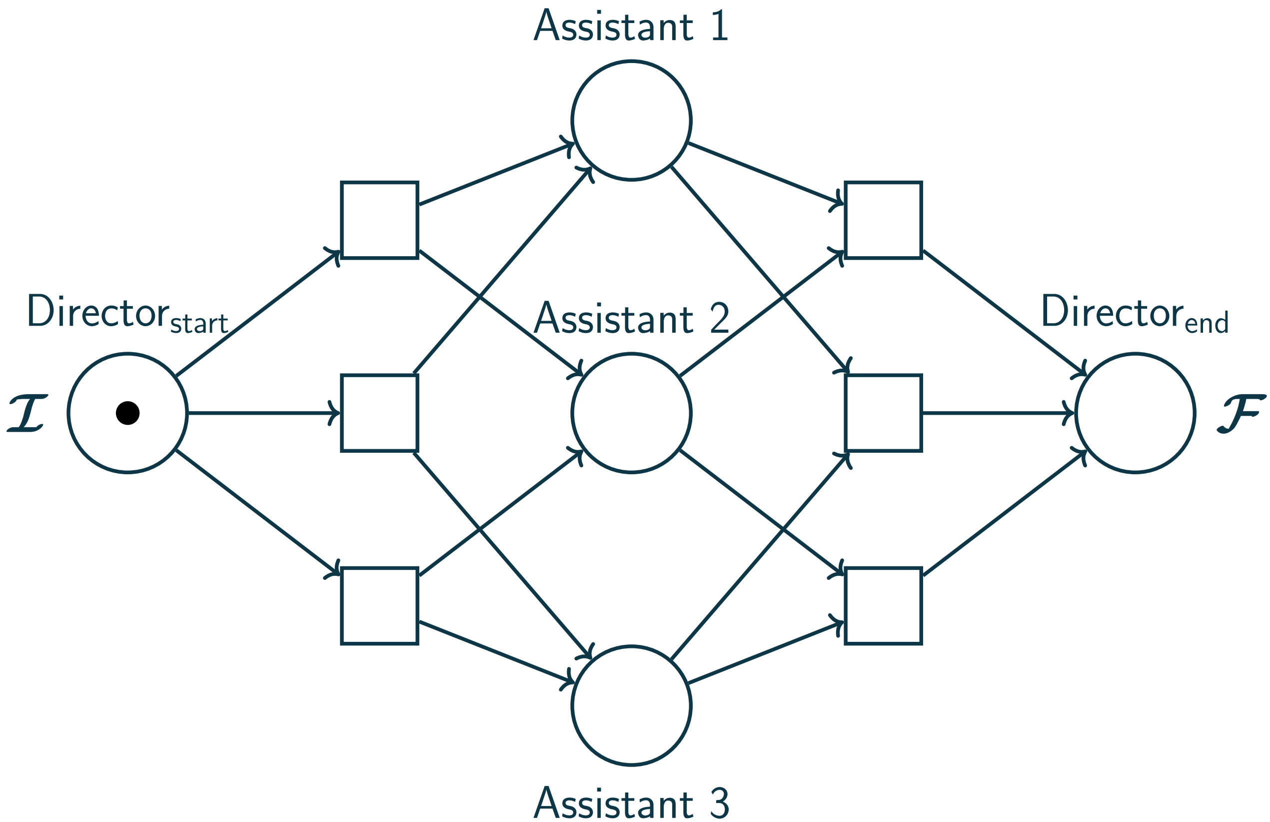

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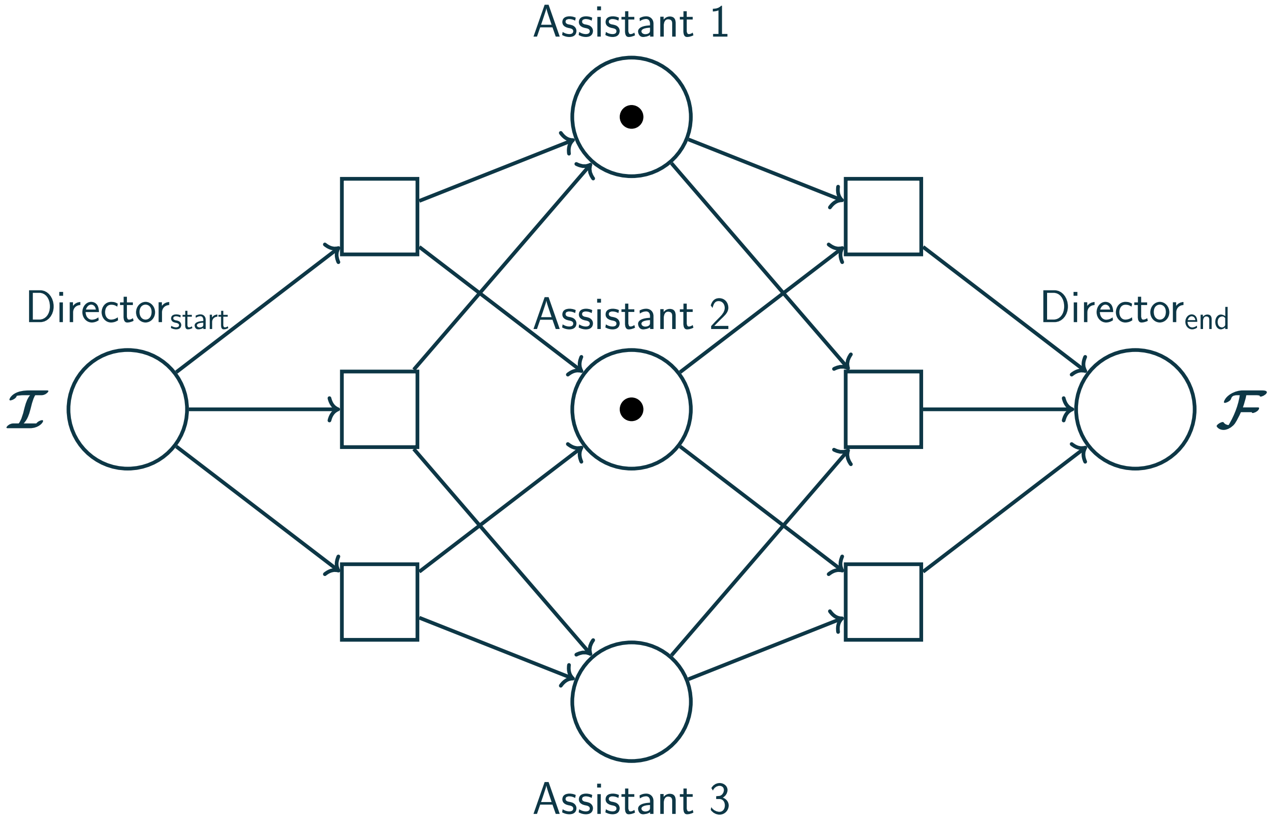

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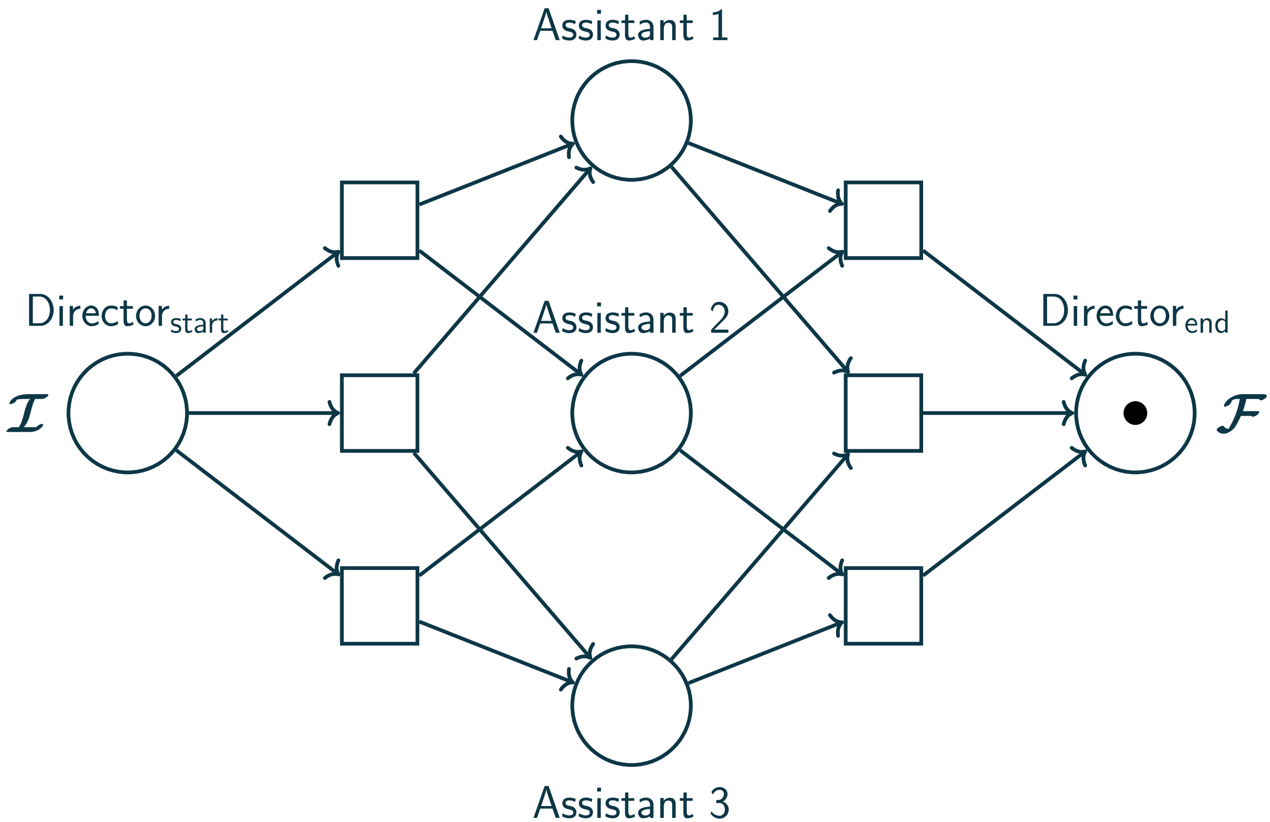
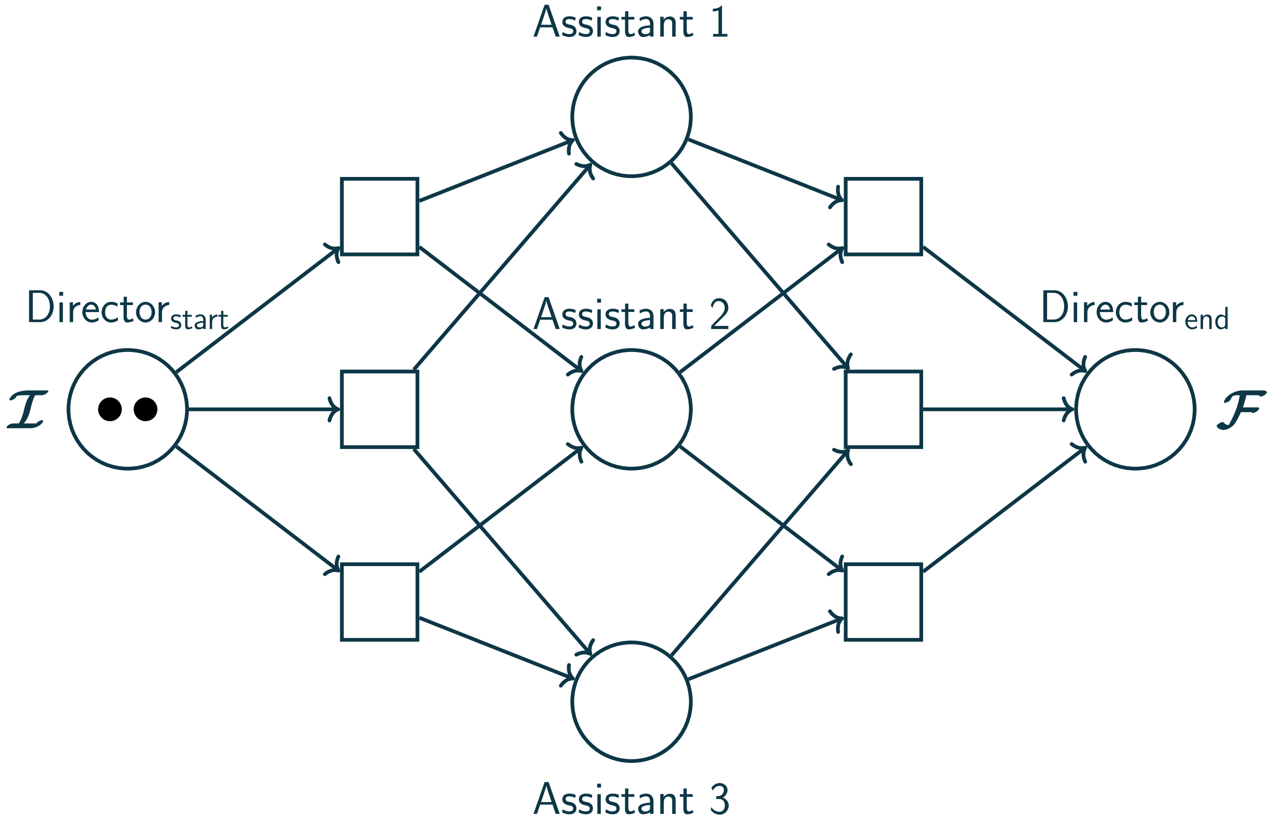


1-sound ✓

Variants of soundness



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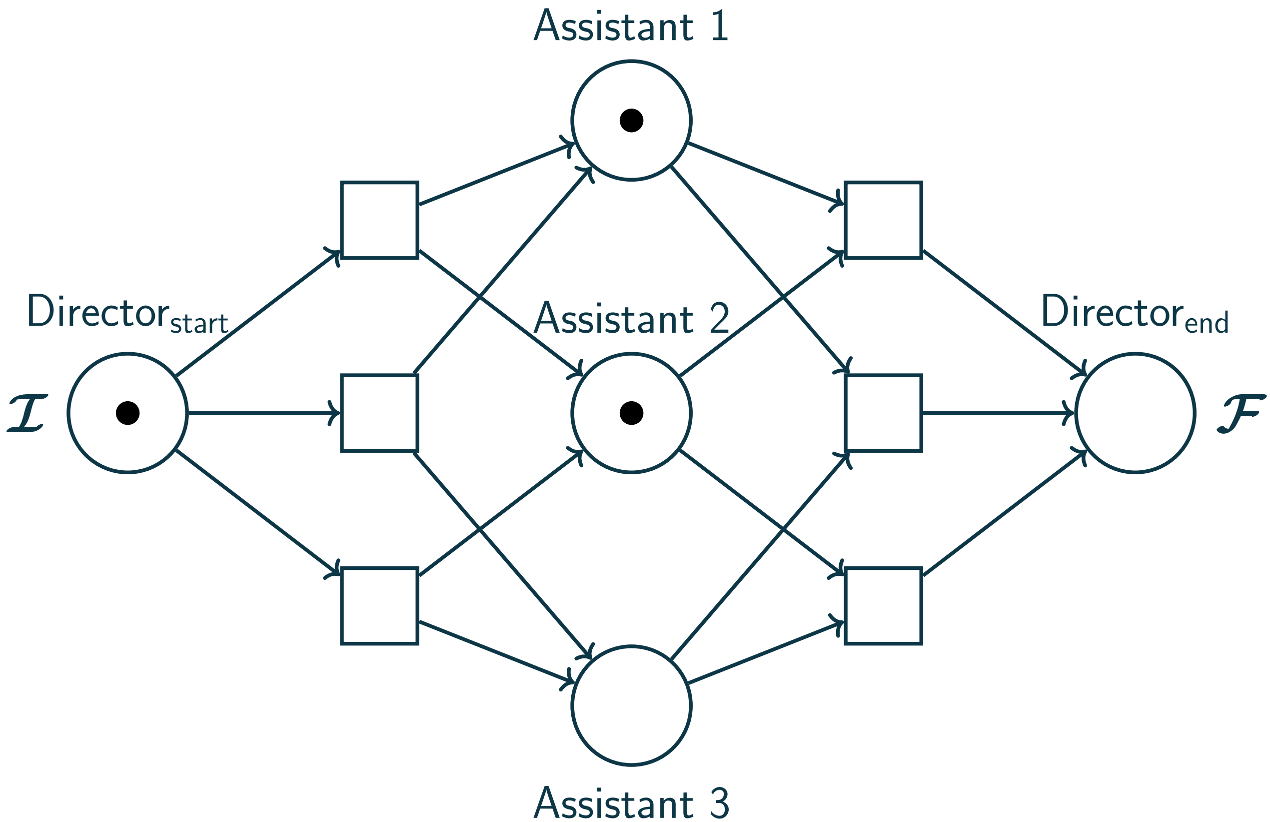

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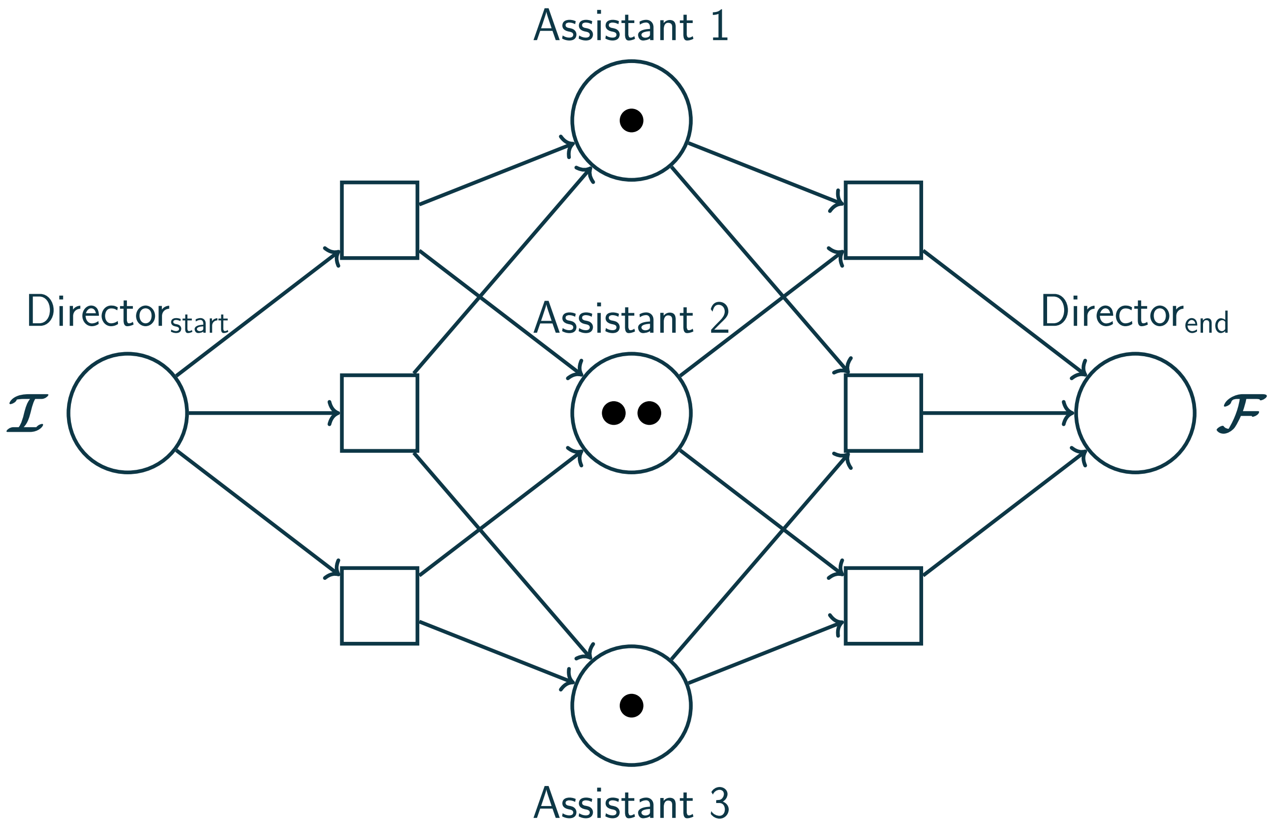

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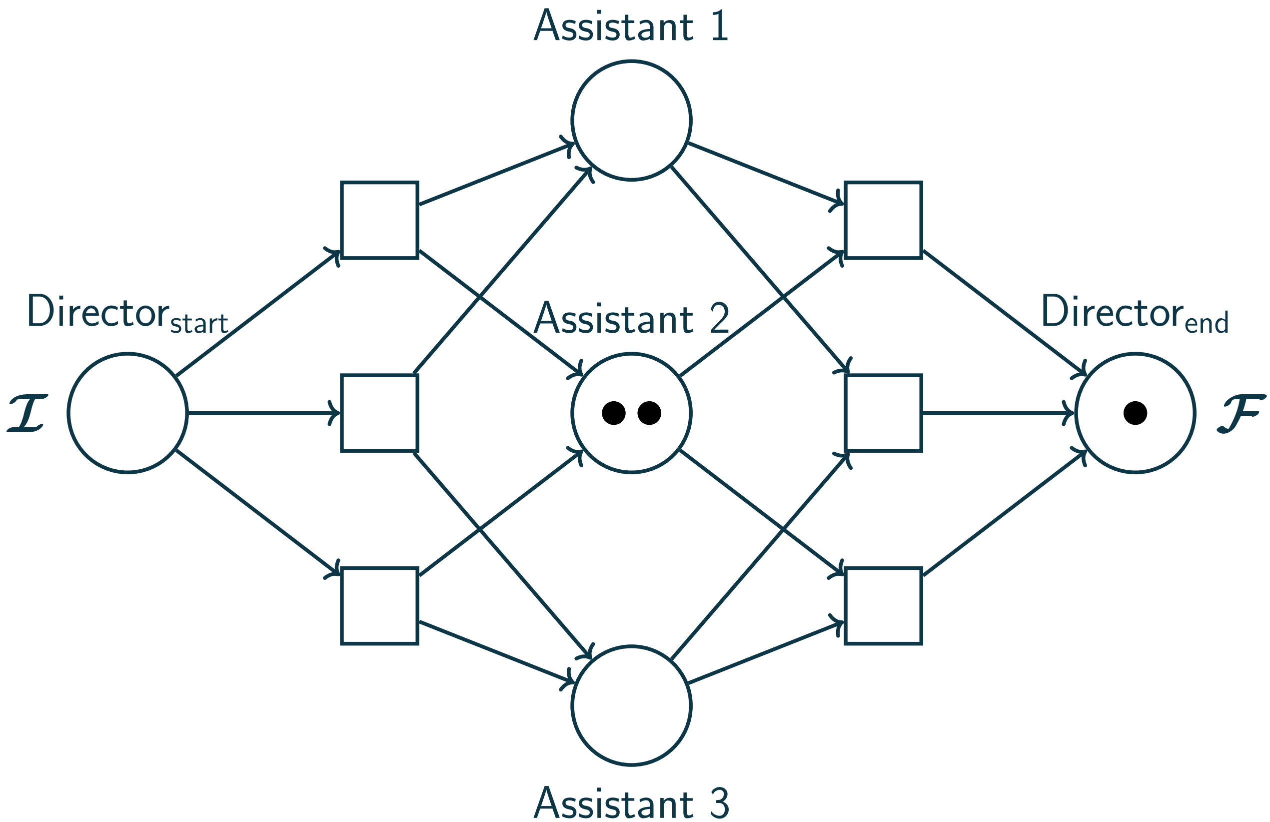

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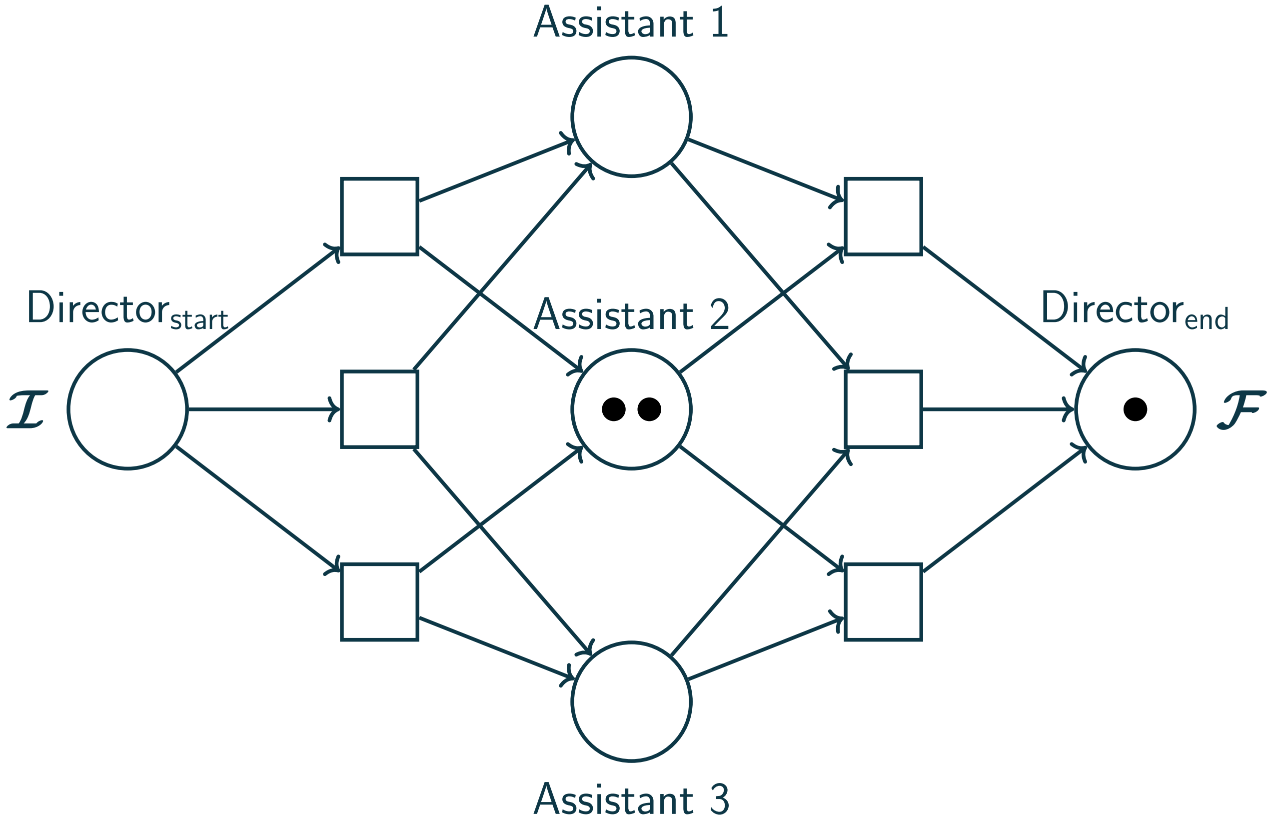

1-sound ✓ 2-sound X

Checking soundness - complexity?

	known results	our work
k-Soundness		
Generalised Soundness		
Structural Soundness		

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[LICS '22]

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[LICS '22]

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N **is** 1**-sound** $\Leftrightarrow$ $(N_{\mathrm{SC}}, \{\mathcal{I} : 1\})$ **is cyclic** $+$ **bounded**

$(N_{\mathrm{SC}}, \{\mathcal{I}\colon 1\})$ **is**

N **is** 1**-sound** $\Leftrightarrow$ **cyclic** $+$ **bounded**

Any reachable marking can reach $\{\mathcal{F}\colon 1\}$

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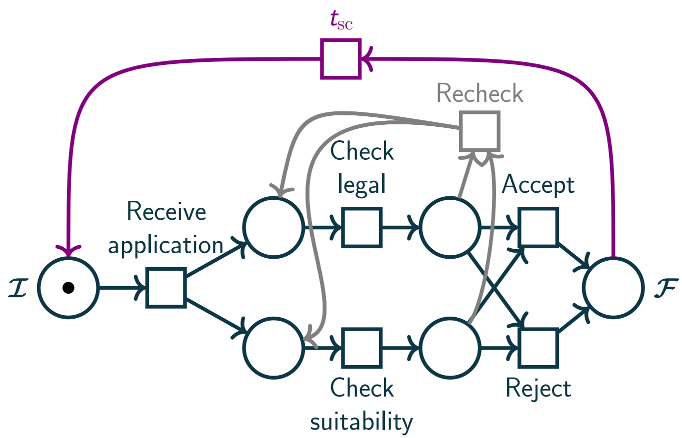

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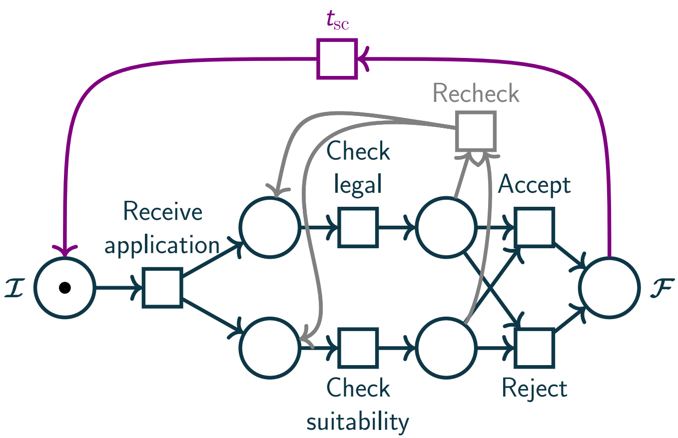

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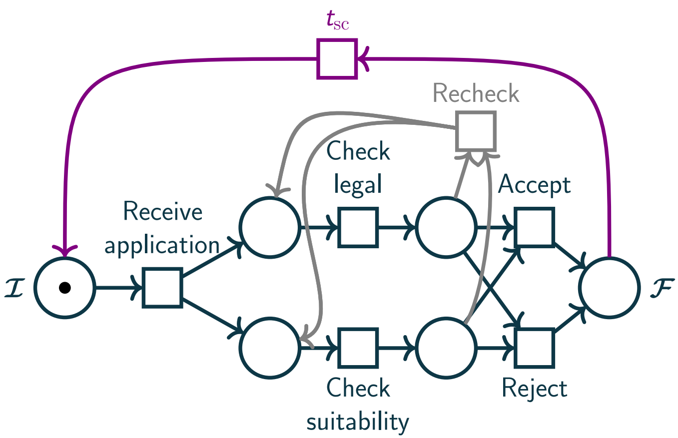

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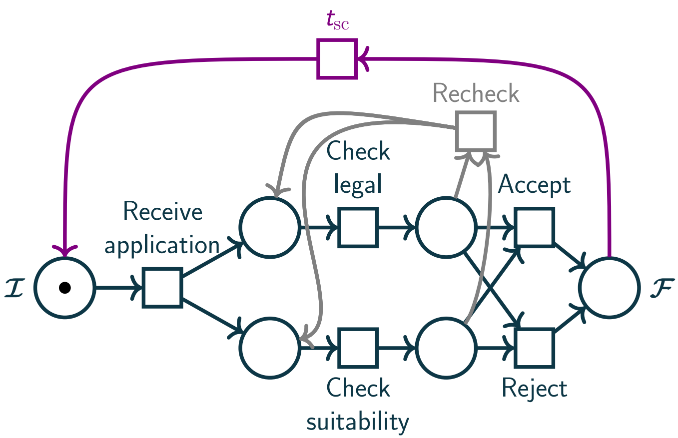

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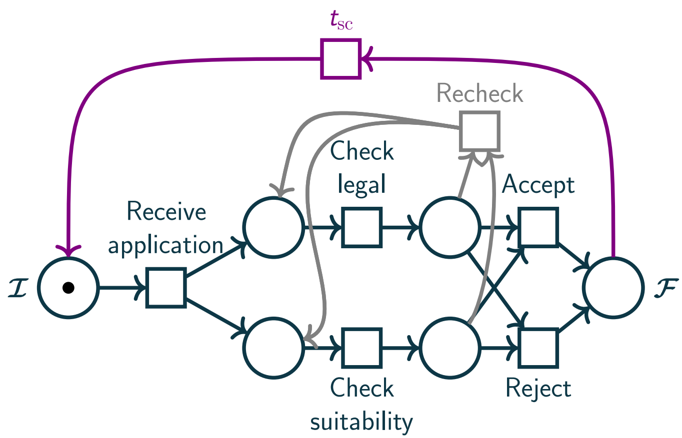
Unbounded: $\{\mathcal{I}: 1\}$ can reach m which can reach m′ with m < m′



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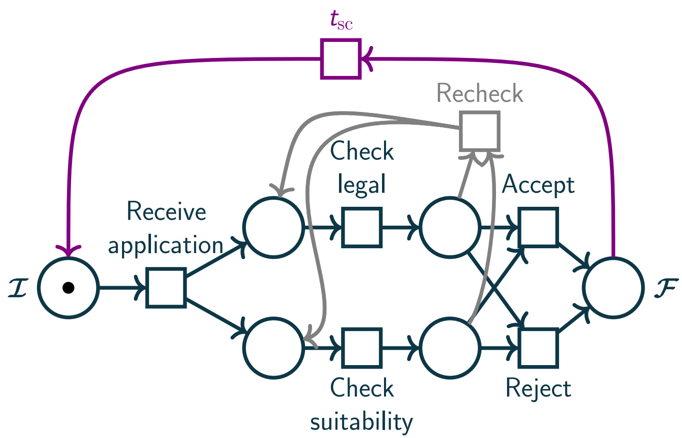

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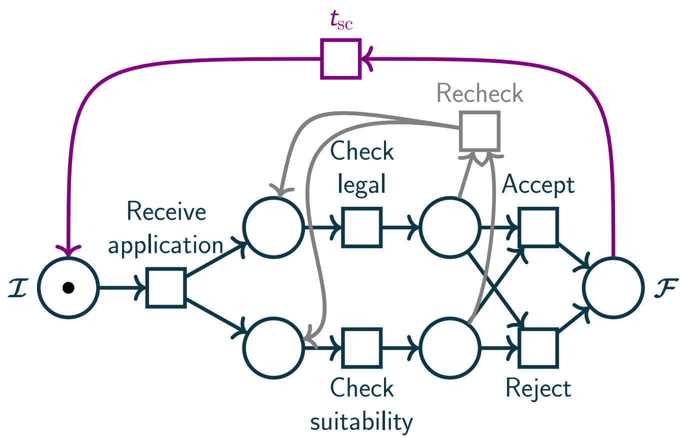
$\{\mathcal{I}\colon 1\}$ reaches m implies m reaches $\{\mathcal{I}\colon 1\}$



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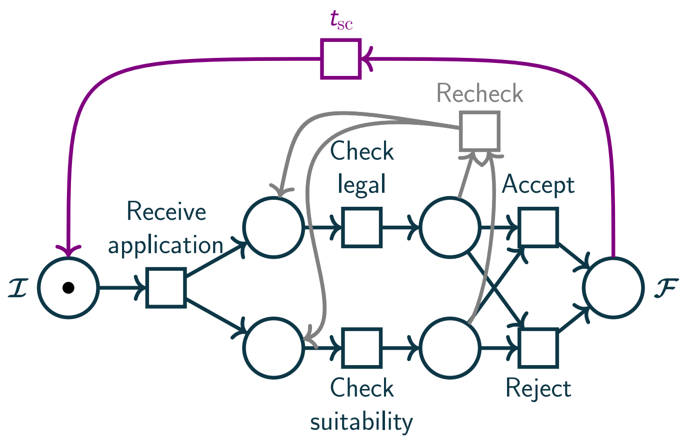
$\{\mathcal{I}\colon 1\}$ reaches m implies m reaches $\{\mathcal{I}\colon 1\}$

$\downarrow$

m reaches m′ which marks $\mathcal{F}$



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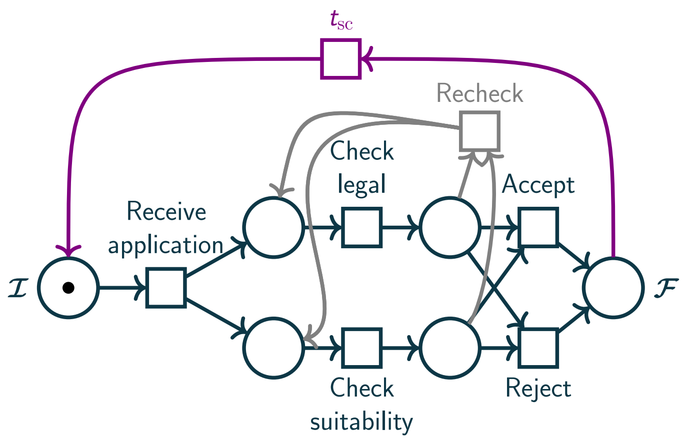

$\{\mathcal{I}\colon 1\}$ reaches m implies m reaches $\{\mathcal{I}\colon 1\}$

$\downarrow$

m reaches m$'$ which marks $\mathcal{F}$

$\downarrow$

m$'$ $= \{\mathcal{F}\colon 1\}$
otherwise N_{SC} is unbounded

$(N_{\mathrm{SC}}, \{\mathcal{I}\colon 1\})$ **is**

N **is** 1-**sound** $\Leftarrow$ **cyclic** + **bounded**

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Any reachable marking can reach $\{\mathcal{I}\colon 1\}$

Unbounded: $\{\mathcal{I}\colon 1\}$ can reach m which can reach m′ with m < m′

$\{\mathcal{I}\colon 1\}$ reaches m implies m reaches $\{\mathcal{I}\colon 1\}$

↓

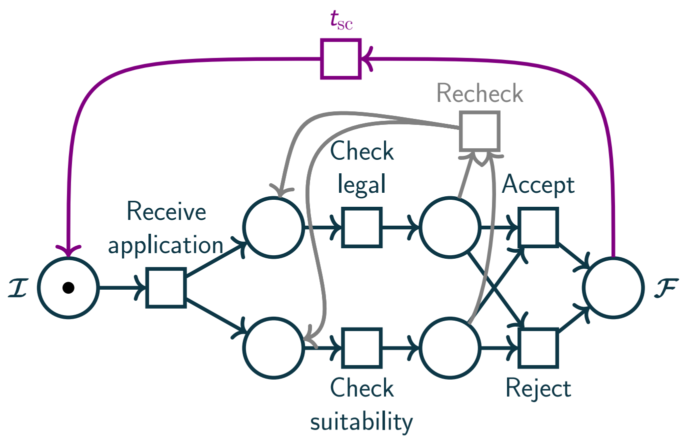
m reaches m′ which marks $\mathcal{F}$

↓

m′ = $\{\mathcal{F}\colon 1\}$
otherwise N_{SC} is unbounded

↓

Any reachable marking can reach $\{\mathcal{F}\colon 1\}$



$$N \textbf{ is 1-sound} \Rightarrow (N_{\mathrm{SC}}, \{\mathcal{I}\colon 1\}) \textbf{ is cyclic} + \textbf{bounded}$$

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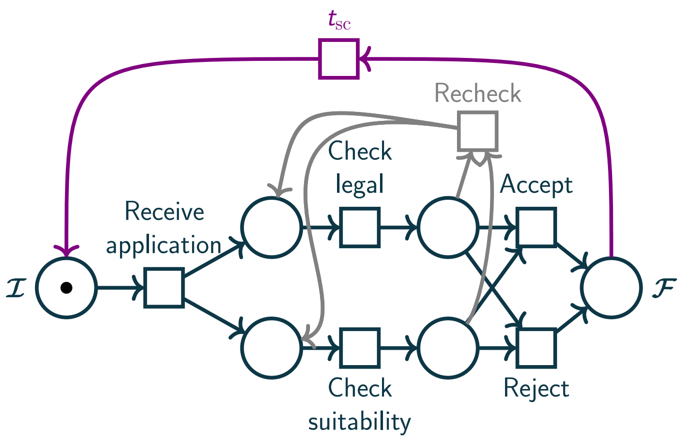

$(N_{\mathrm{SC}}, \{\mathcal{I}\colon 1\})$ **is**

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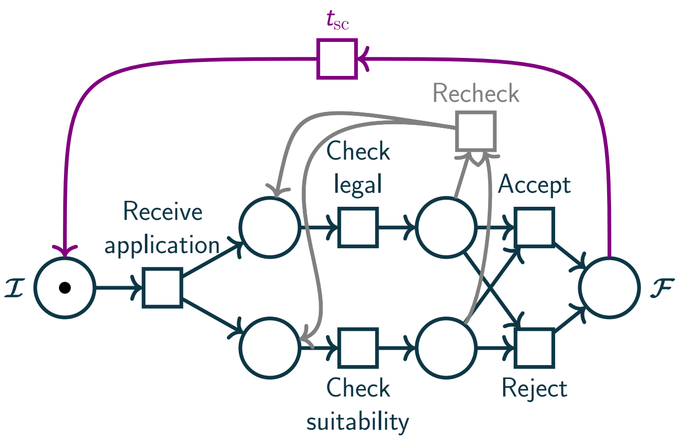

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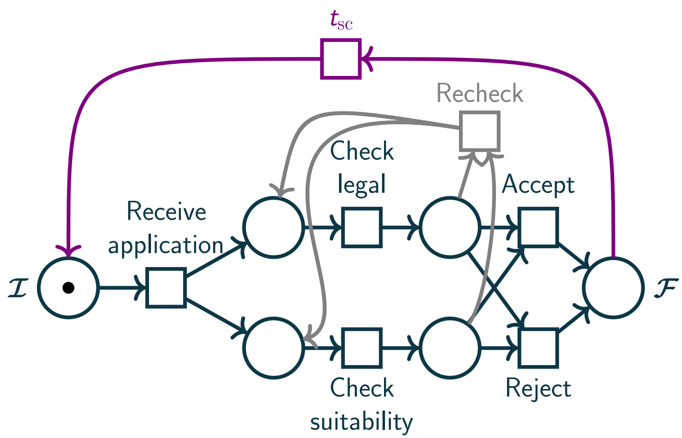

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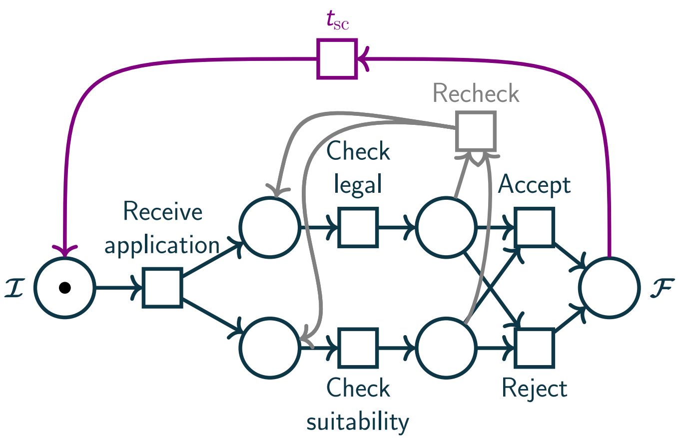

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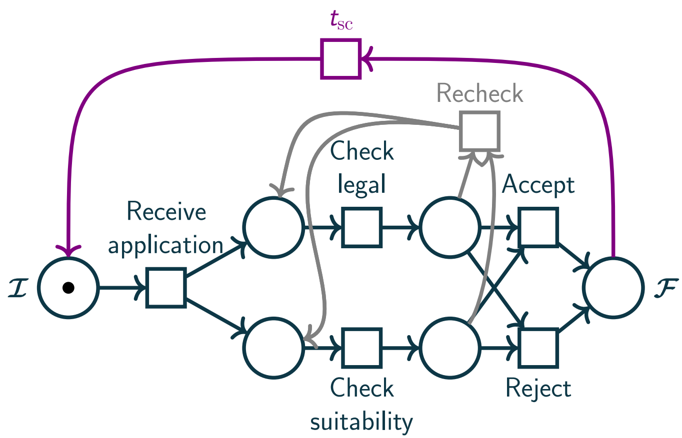
Assume N_{SC} is unbounded but N is 1-sound



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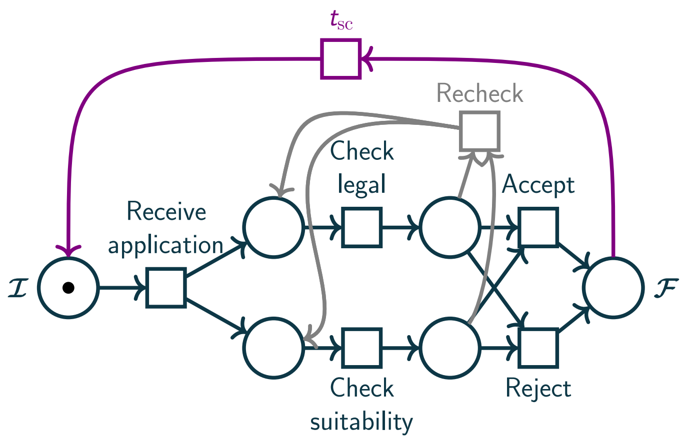
$\{\mathcal{I}\colon 1\}$ can reach m which can reach m$' >$ m: m$'$ = m + n



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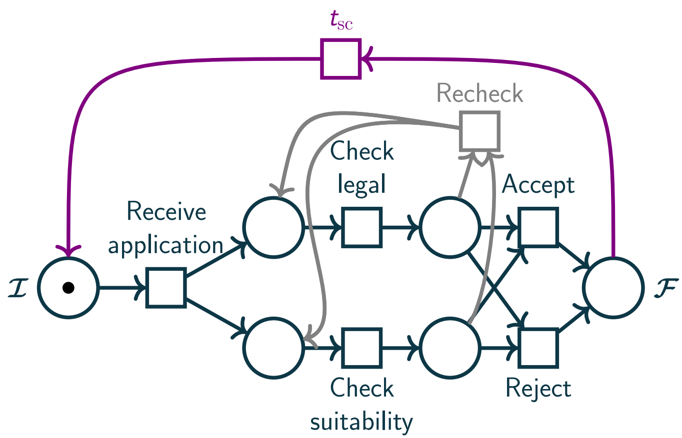

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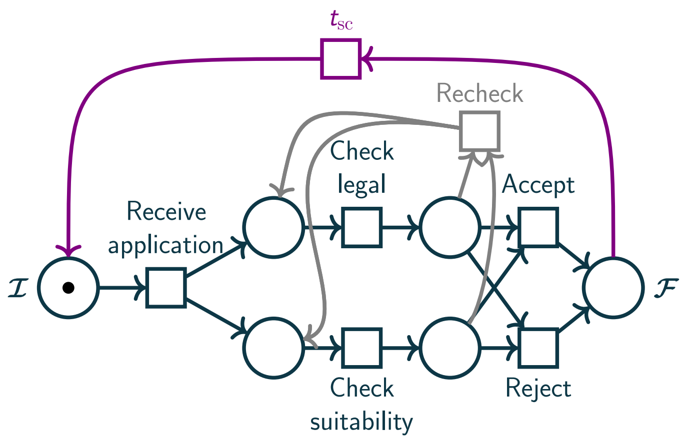

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↓

m can reach $\{\mathcal{F}\colon 1\}$

$\mathrm{m} + \mathrm{n}$ can reach $\{\mathcal{F}\colon 1\} + \mathrm{n}$

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$\{\mathcal{F}\colon 1\} + \mathrm{n}$ cannot reach $\{\mathcal{F}\colon 1\}$

$\Rightarrow N$ is not 1-sound!

N is 1-sound $\Leftrightarrow$ $(N_{\mathrm{SC}}, \{\mathcal{I}\colon 1\})$ is cyclic + bounded

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EXPSPACE-hardness is by reduction from reachability in **reversible Petri nets**

Checking soundness - complexity?

	known results	our work	
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Exact algorithms are impractical in general; instead:

- Focus on semi-decision procedures - *Continuous Soundness* **4.**
  co-NP complete necessary condition for generalised soundness
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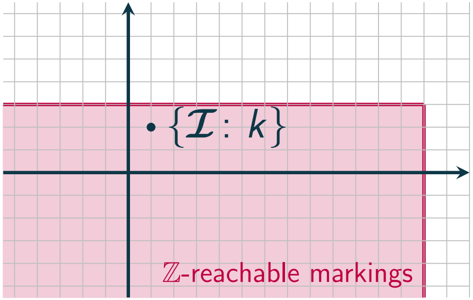

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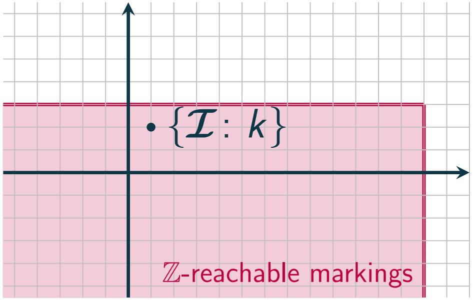

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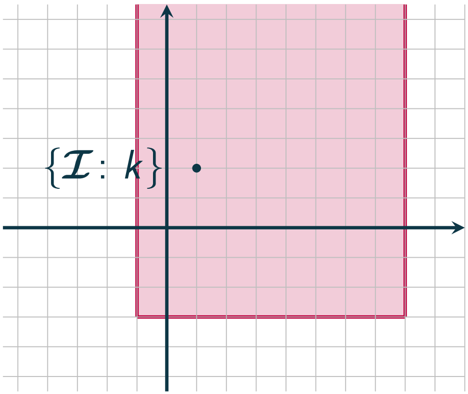

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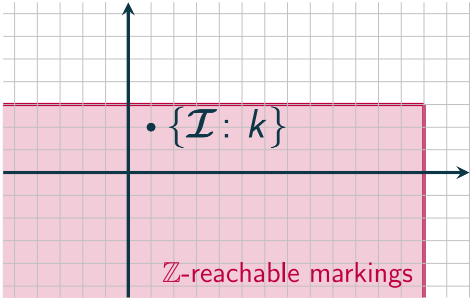

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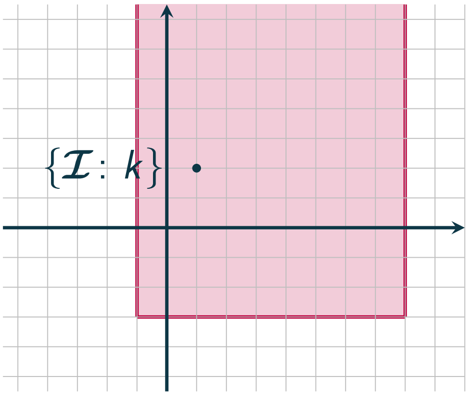

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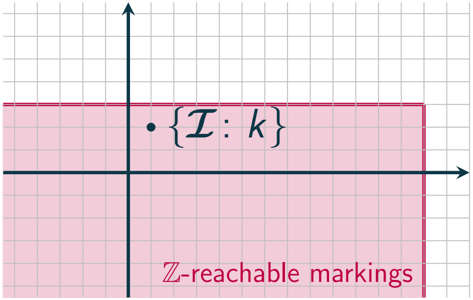

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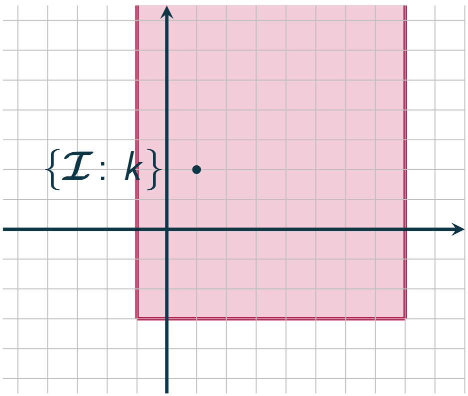

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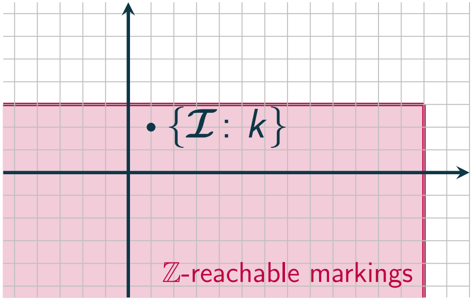

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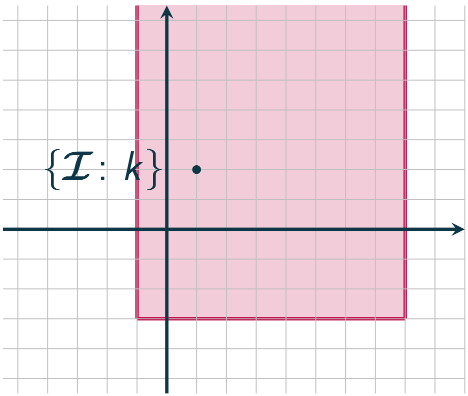

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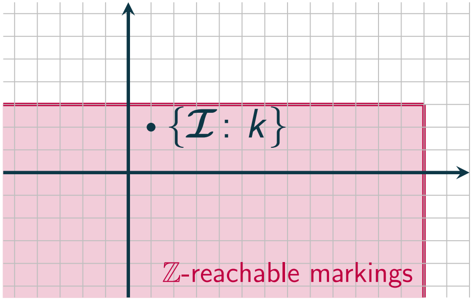

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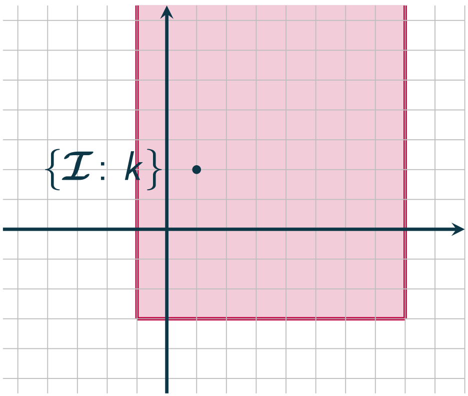

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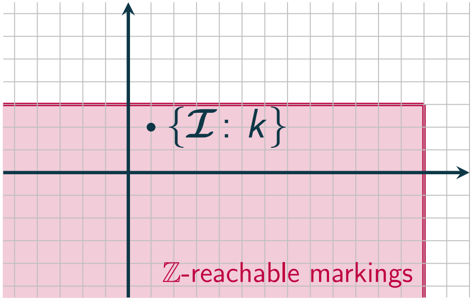

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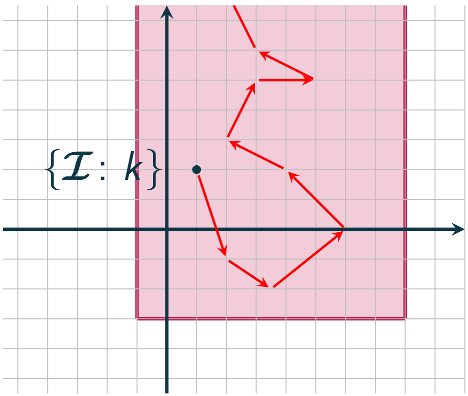

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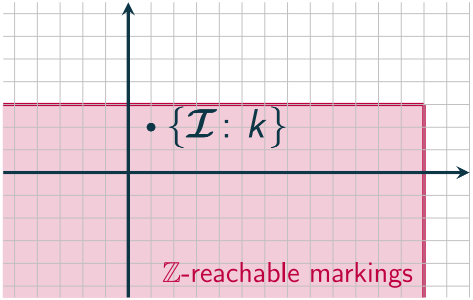

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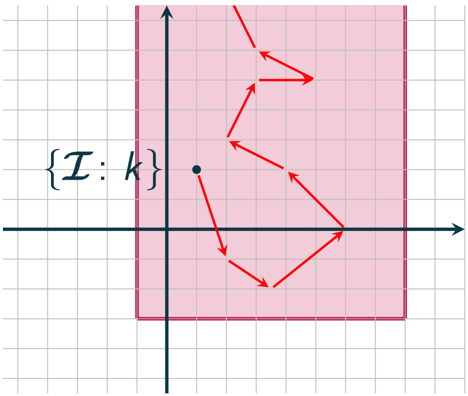

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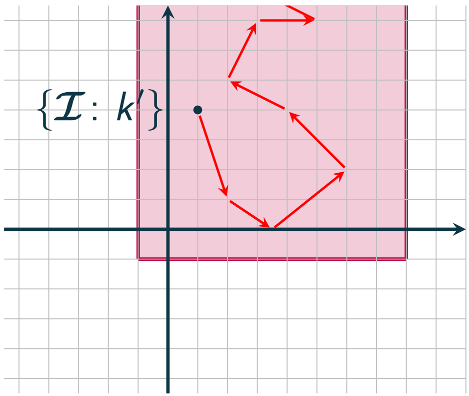

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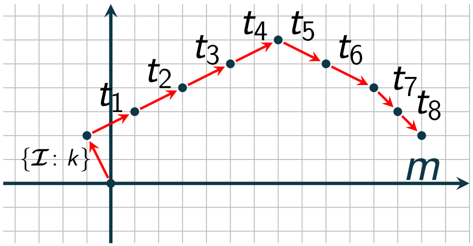

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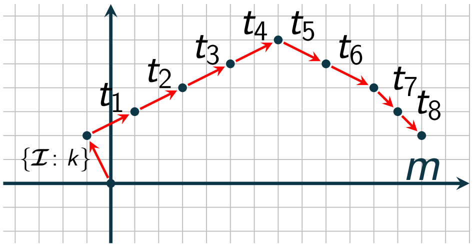

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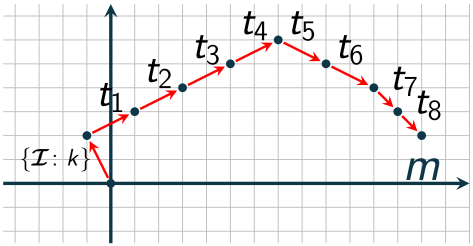

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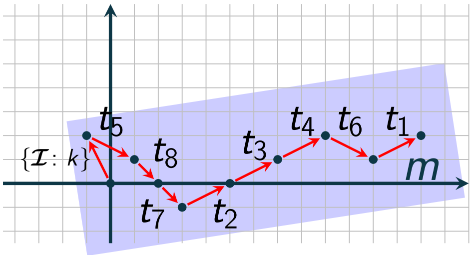

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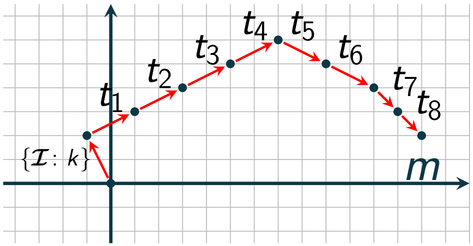

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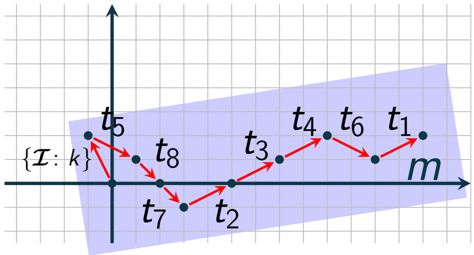

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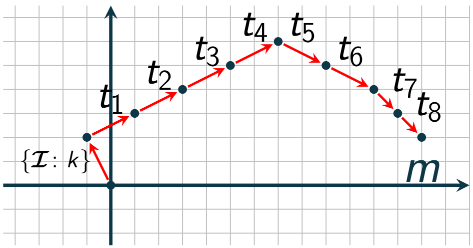
Big reachable markings imply $\mathbb{Z}$-unboundedness

$\{\mathcal{I}\colon k\} \xrightarrow{\text{very large}} m$ Big markings must be reached by long runs



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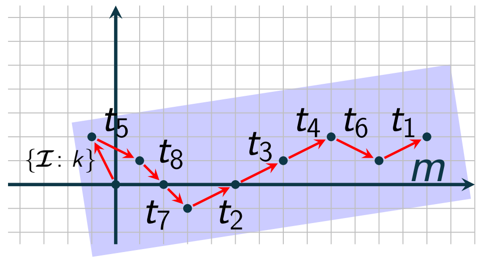

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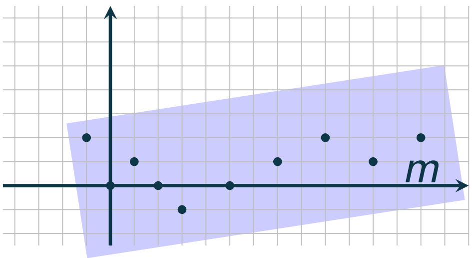

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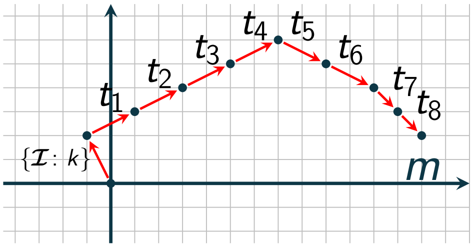

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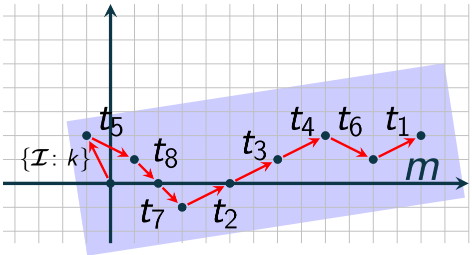

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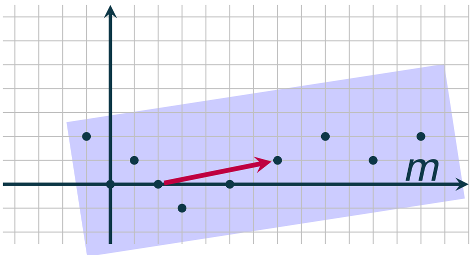

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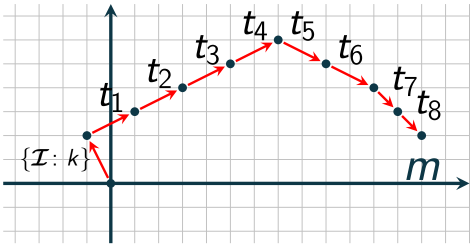

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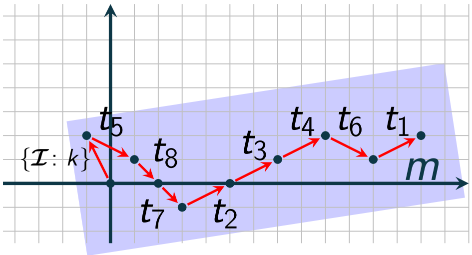

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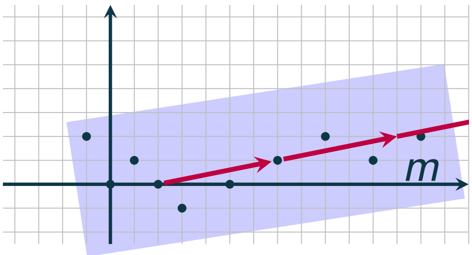

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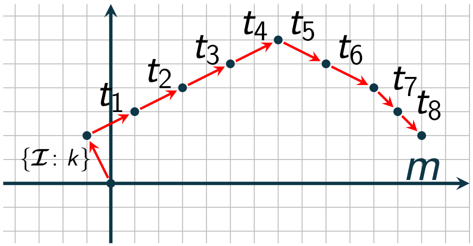

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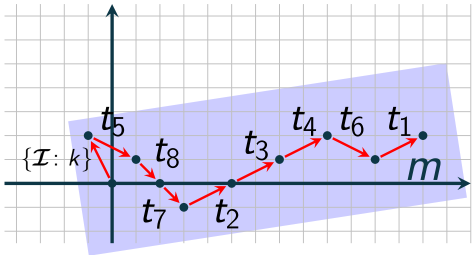

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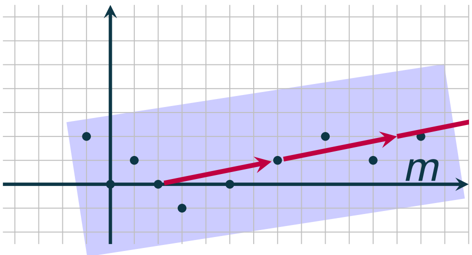

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Generalised soundness is in PSPACE

N is **generalised sound:**
$\forall k : \{\mathcal{I}: k\} \to m$ implies $m \to \{\mathcal{F}: k\}$

Witness k's are small: Not generalised sound $\Rightarrow$ unsound for a small k

A helpful necessary condition: Not $\mathbb{Z}$-bounded $\Rightarrow$ not generalised sound **1.**

Only enumerate small markings: Big marking reachable $\Rightarrow$ not $\mathbb{Z}$-bounded **2.**

Algorithm:

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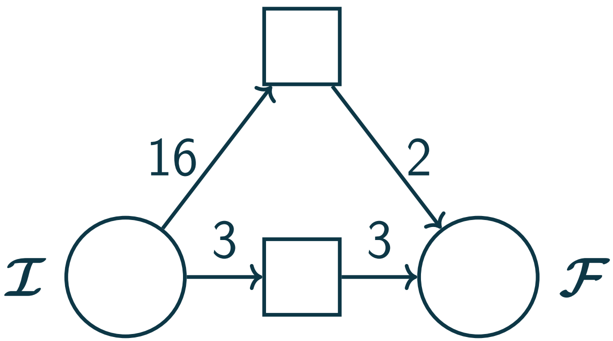

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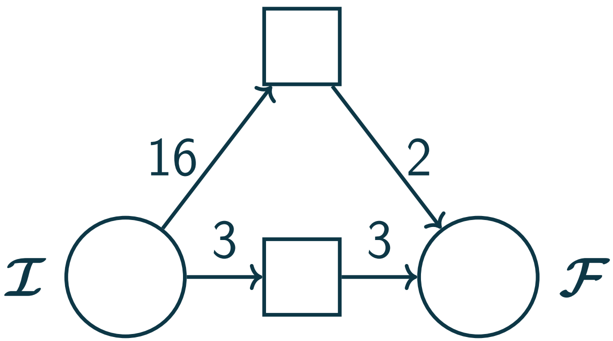

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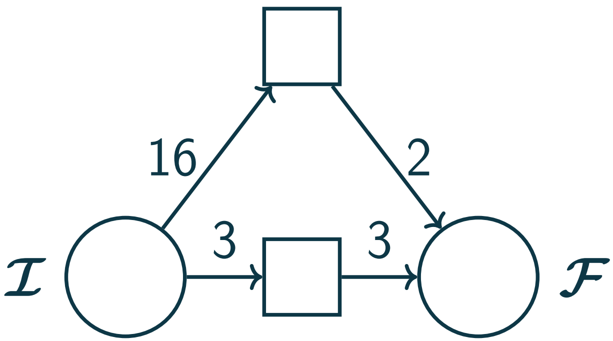

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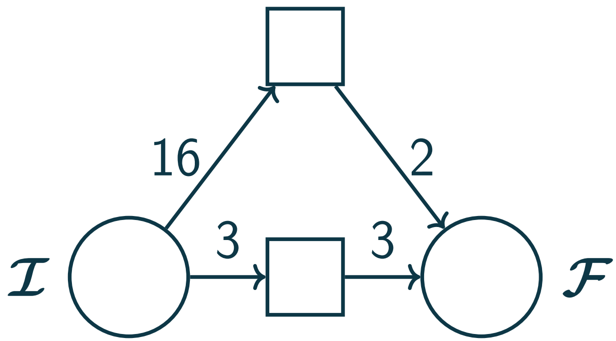

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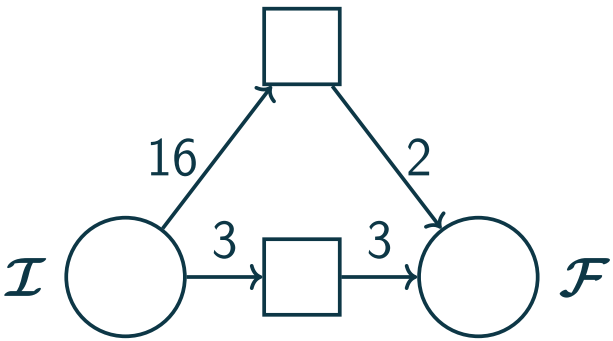

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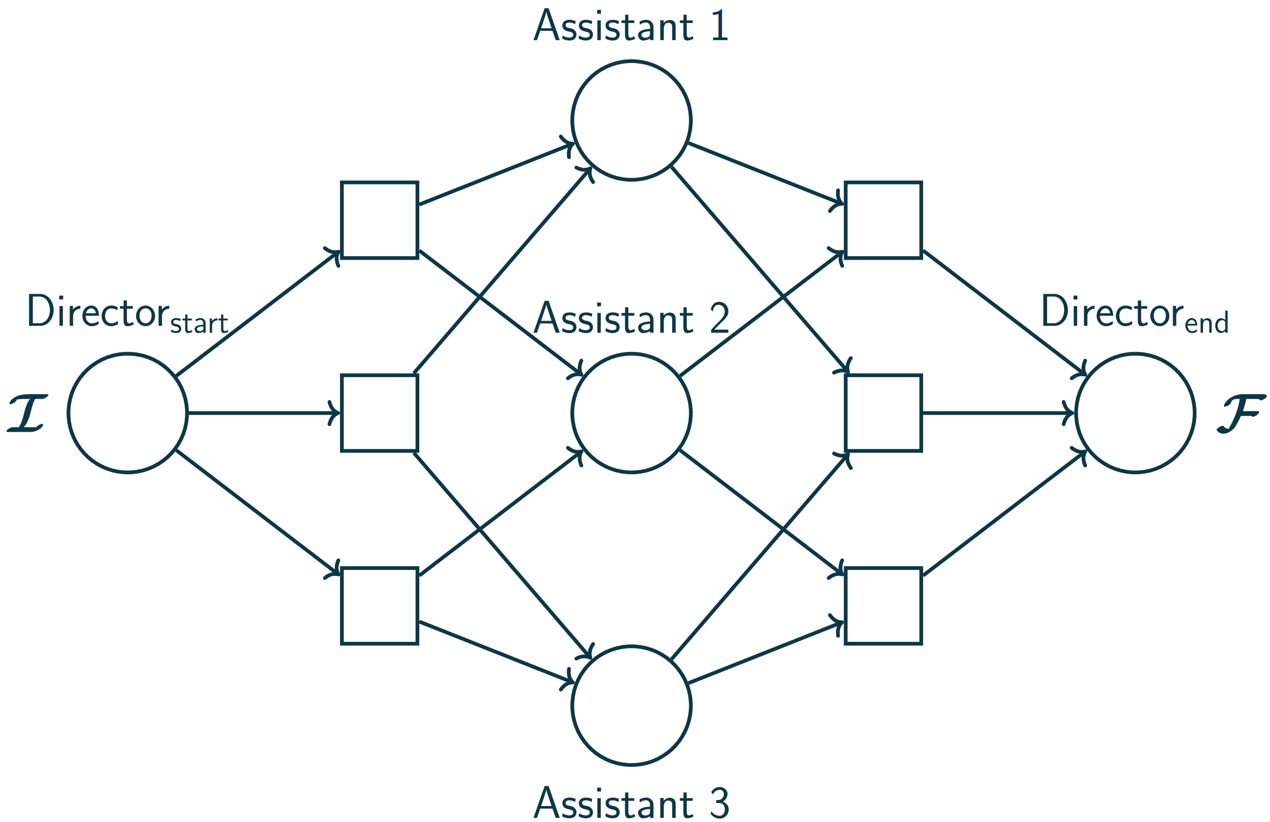

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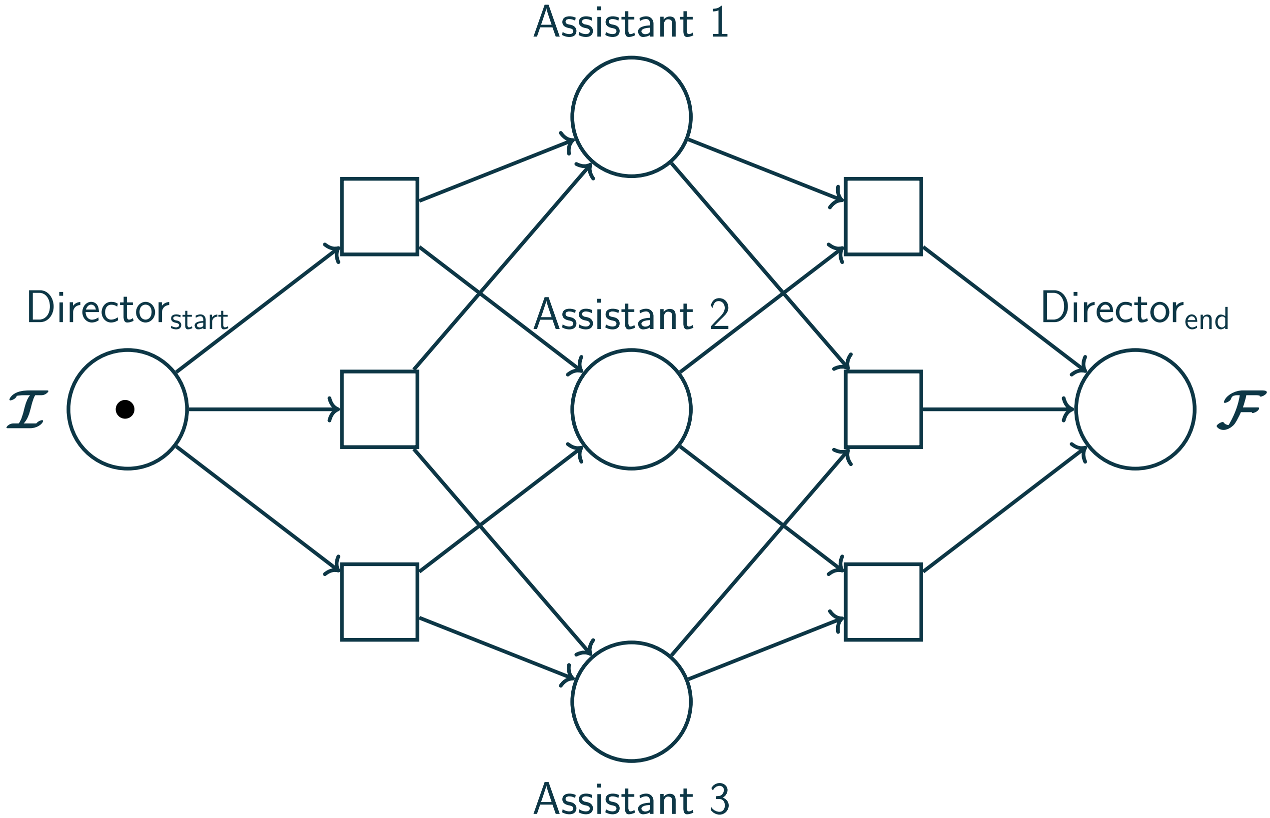

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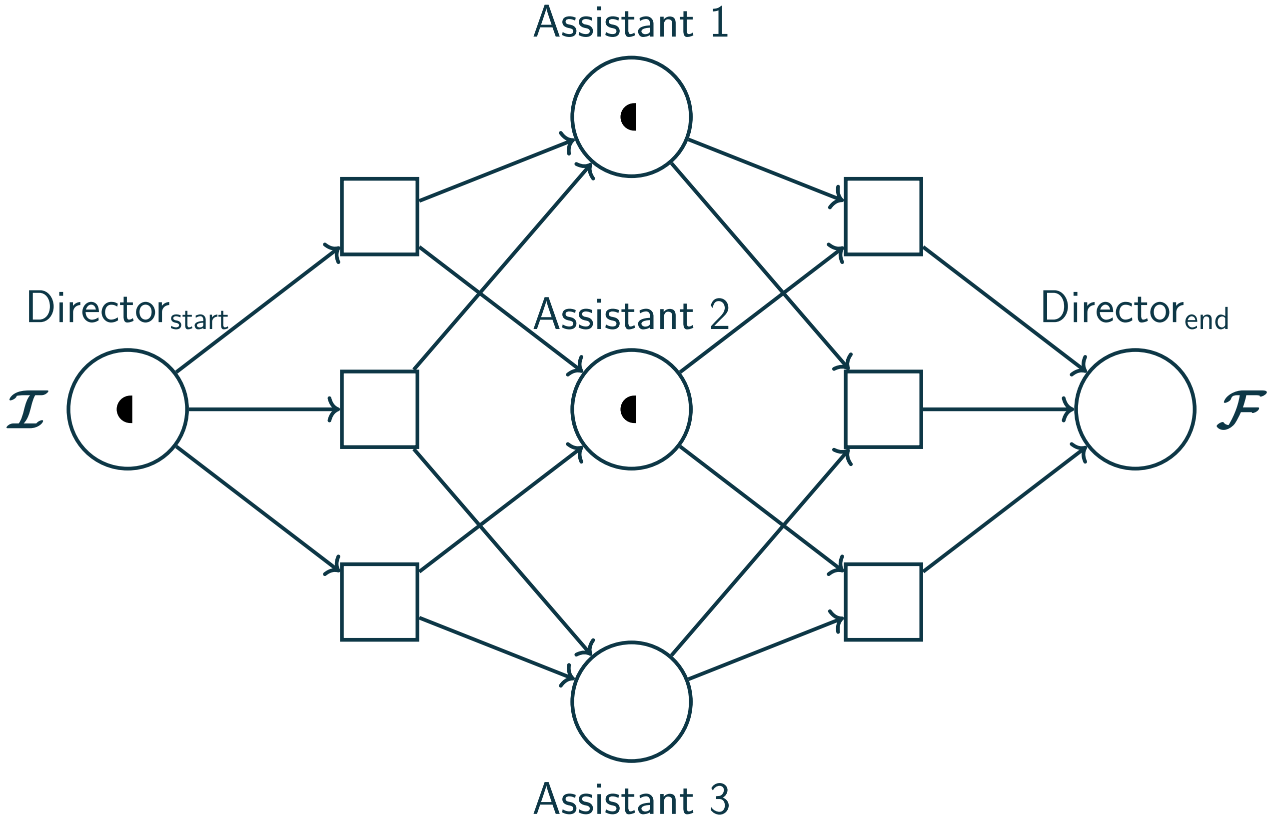

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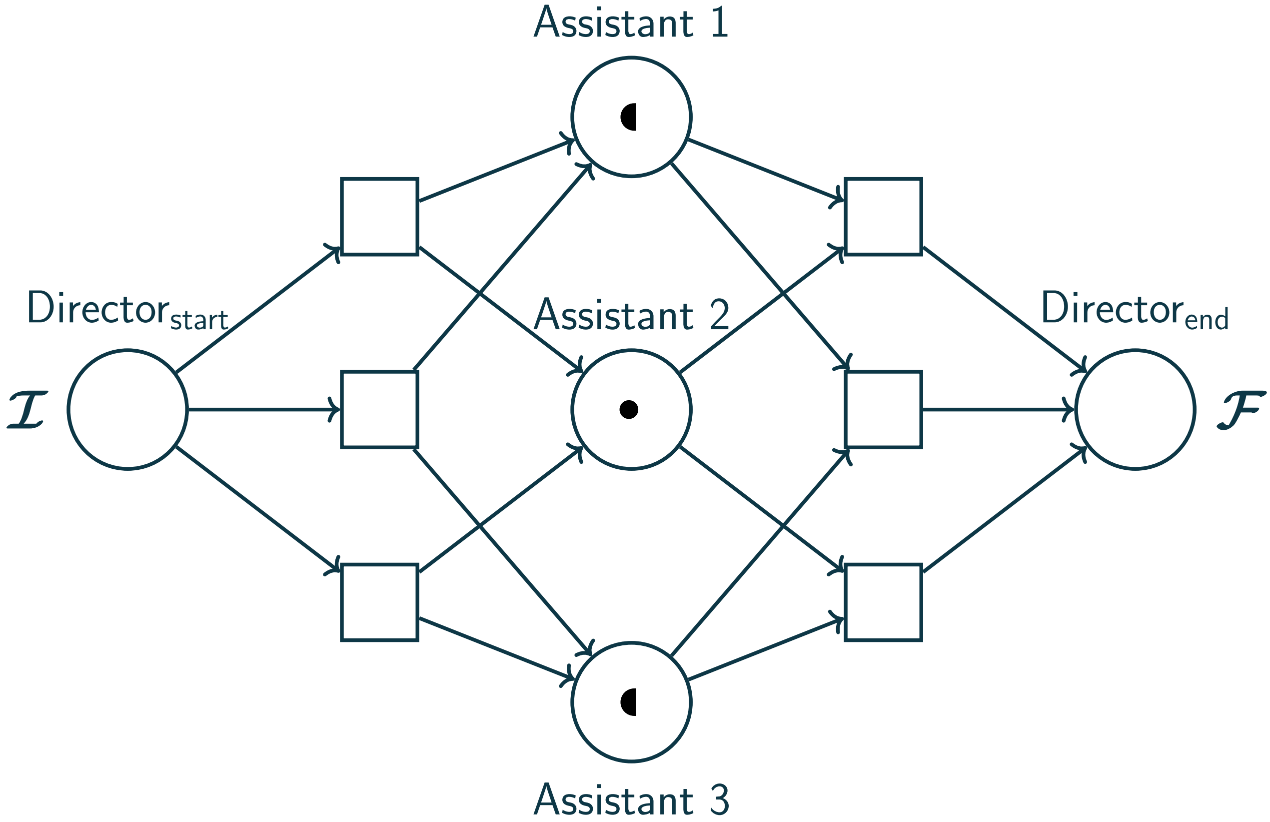

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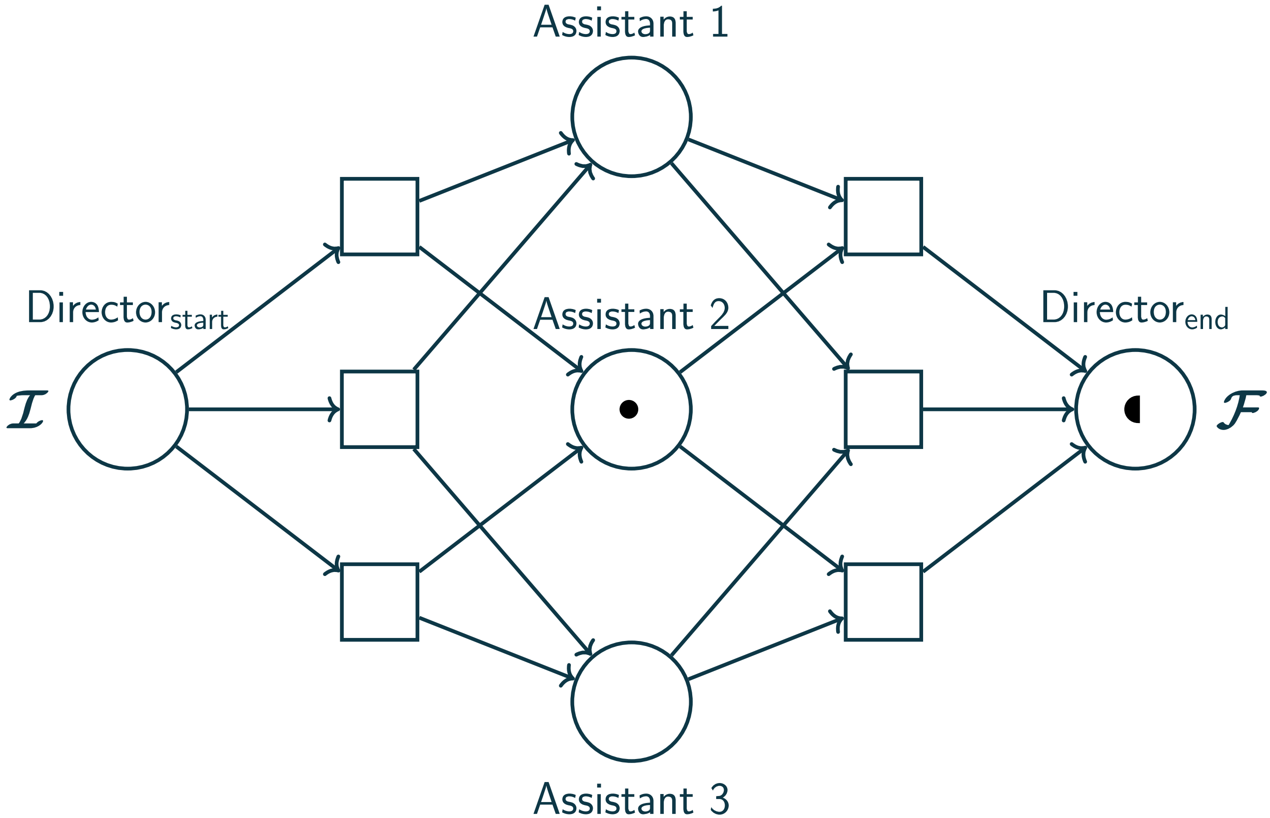

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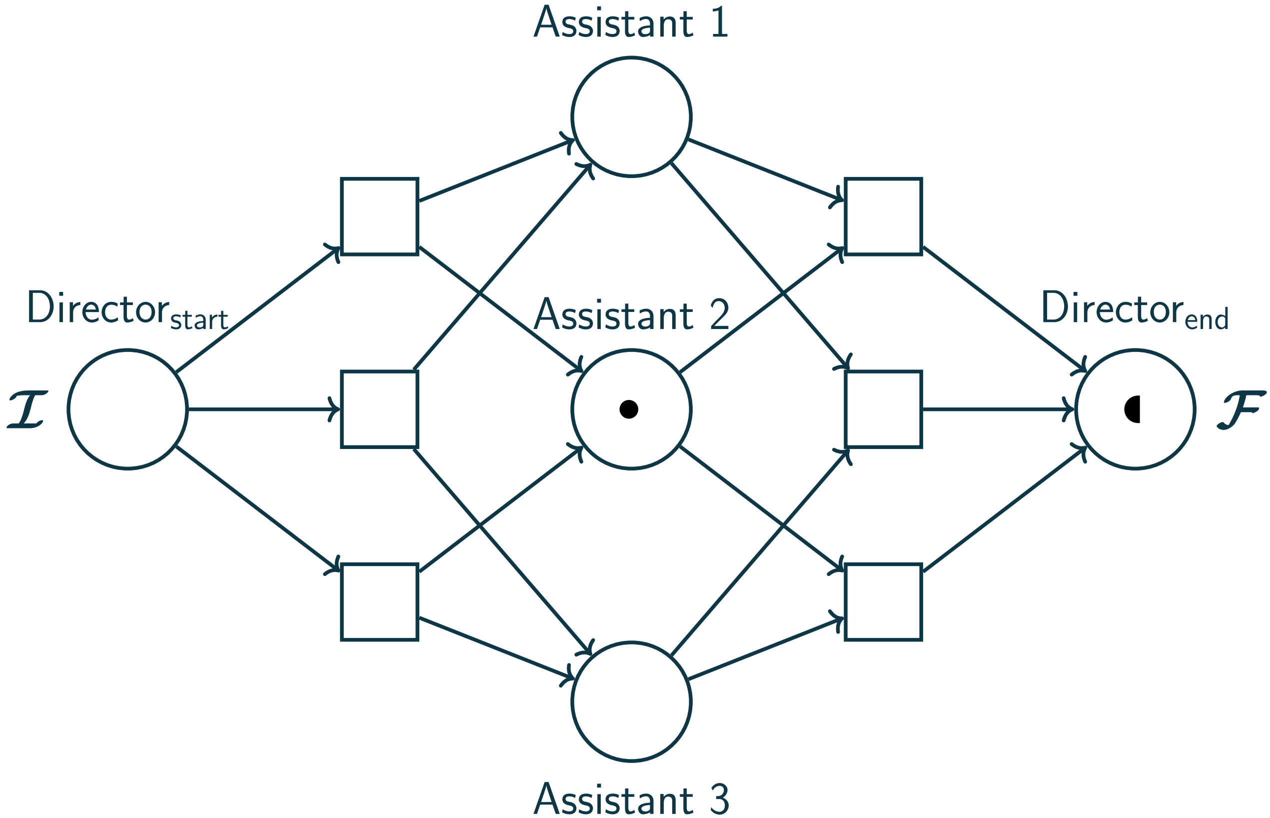

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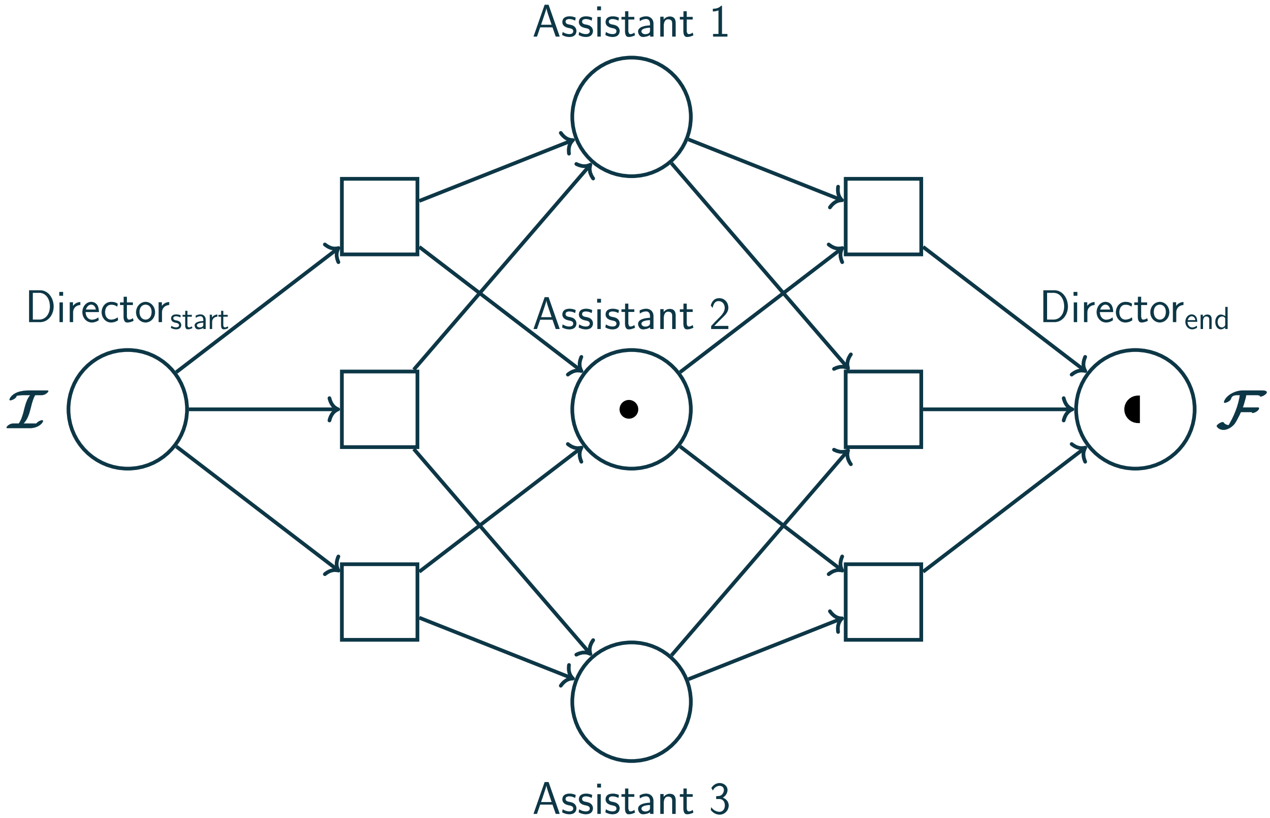

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Advantage: Continuous reachability is in Ptime

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Why does generalised soundness require continuous soundness?

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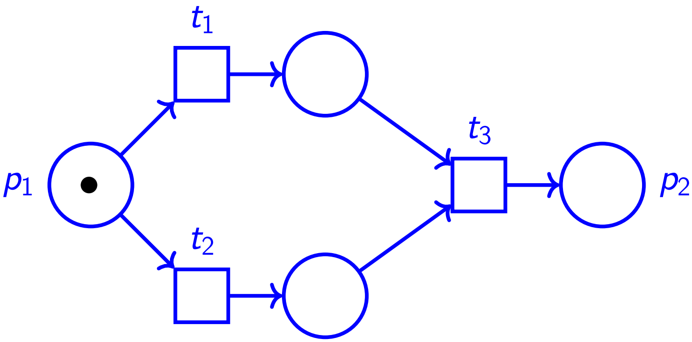

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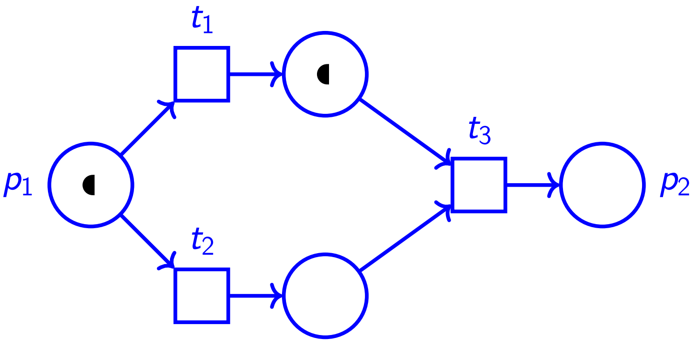

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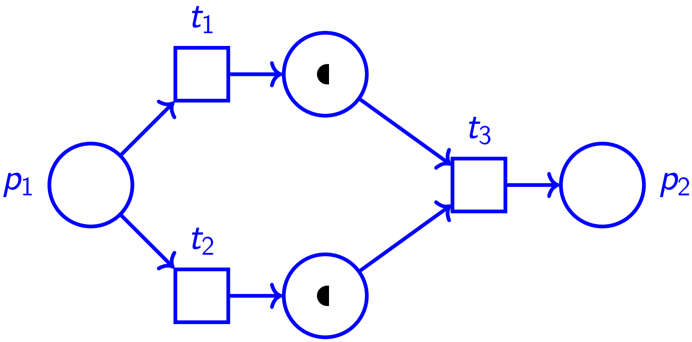

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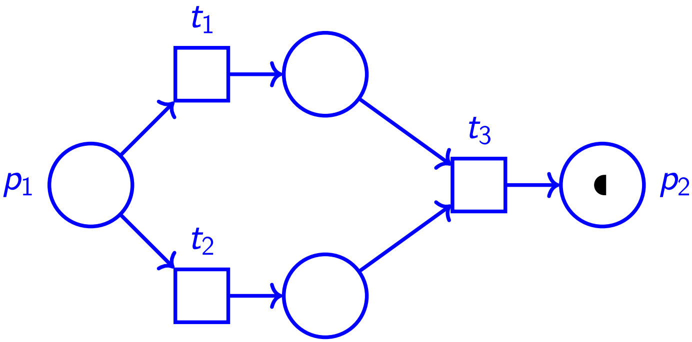

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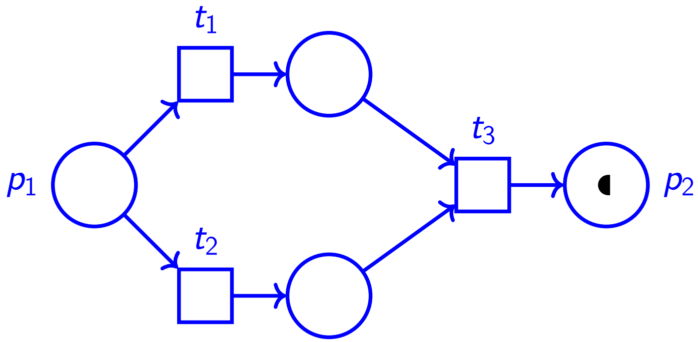

$$\{p_1 : 1\} \xrightarrow{{}^1\!/\!{}_2 t_1 \quad {}^1\!/\!{}_2 t_2 \quad {}^1\!/\!{}_2 t_3} \{p_2 : {}^1\!/\!{}_2\}$$

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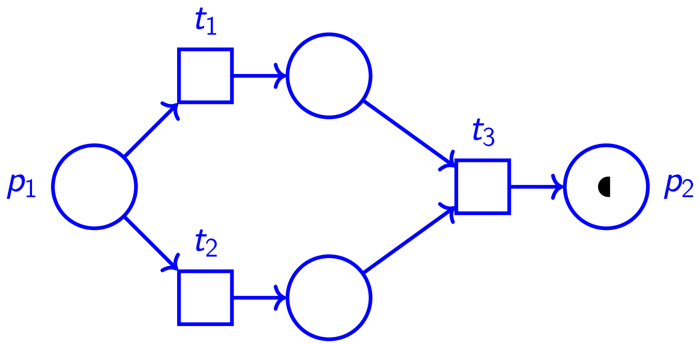

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Continuous reachability

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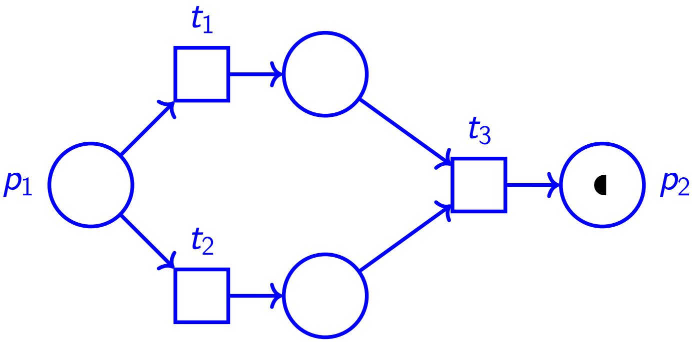

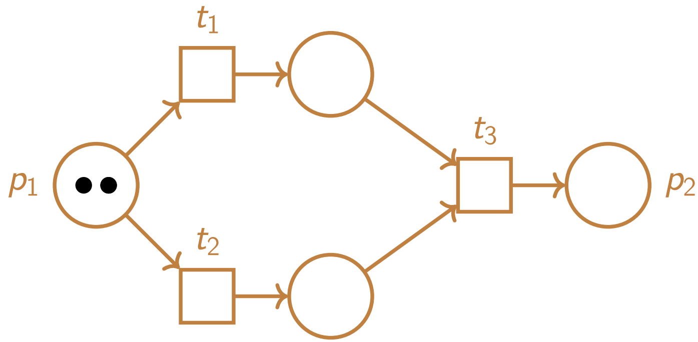

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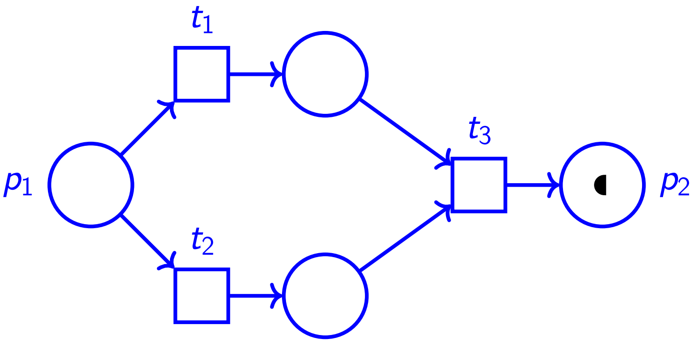

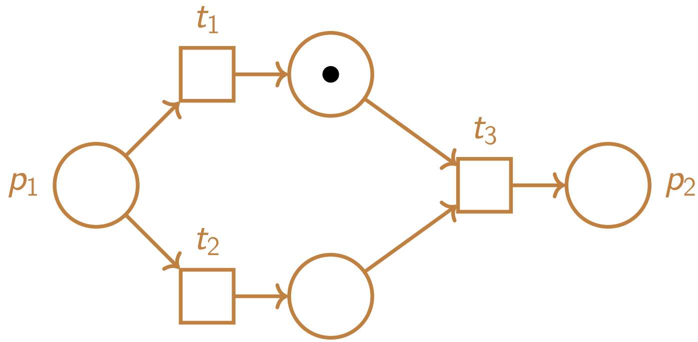

$$\{p_1 : 1\} \xrightarrow{{}^1/_2 t_1 \ \ {}^1/_2 t_2 \ \ {}^1/_2 t_3} \{p_2 : {}^1/_2\}$$

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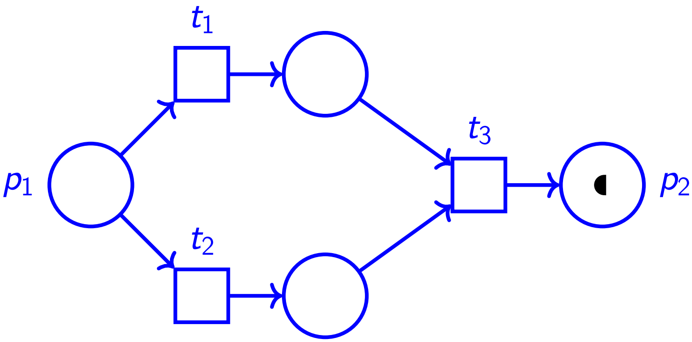

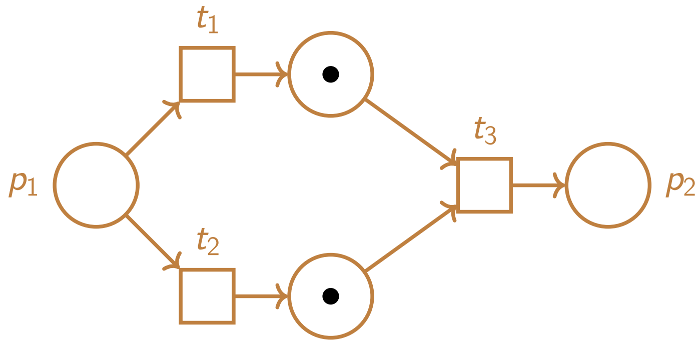

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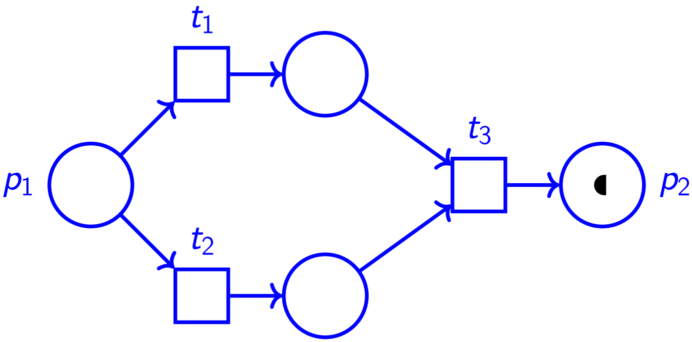

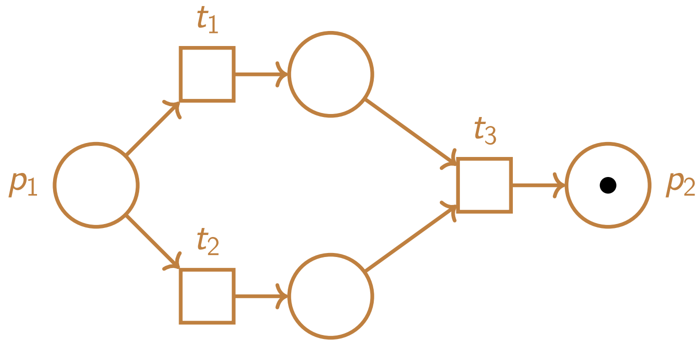

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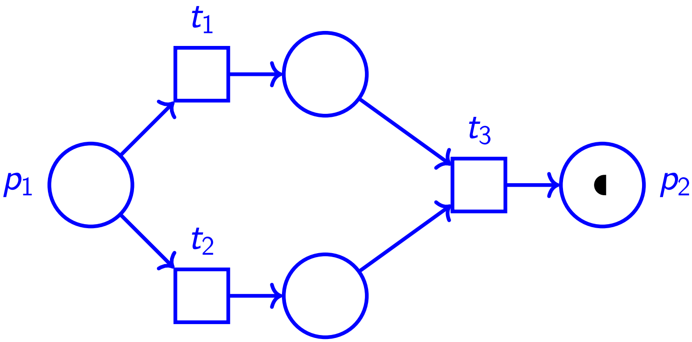

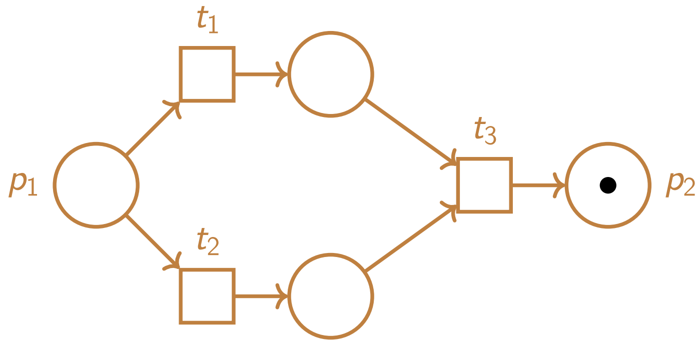

$\{p_1 : 1\} \xrightarrow{1/2 t_1 \;\; 1/2 t_2 \;\; 1/2 t_3} \{p_2 : 1/2\}$

Continuous reachability

$\{p_1 : 2\} \xrightarrow{t_1 \;\; t_2 \;\; t_3} \{p_2 : 1\}$

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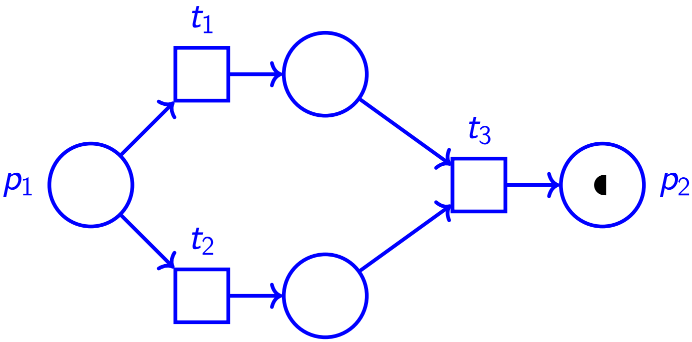

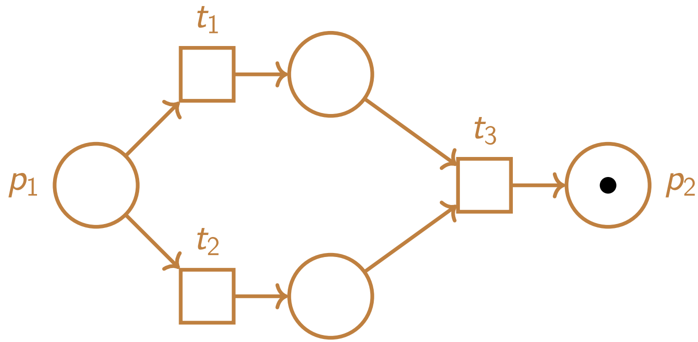

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Reachability with many tokens

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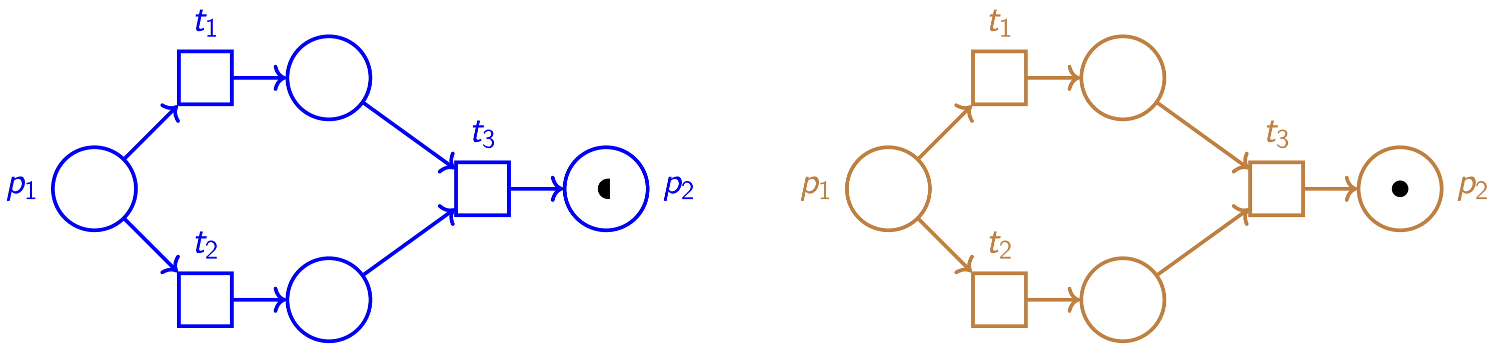

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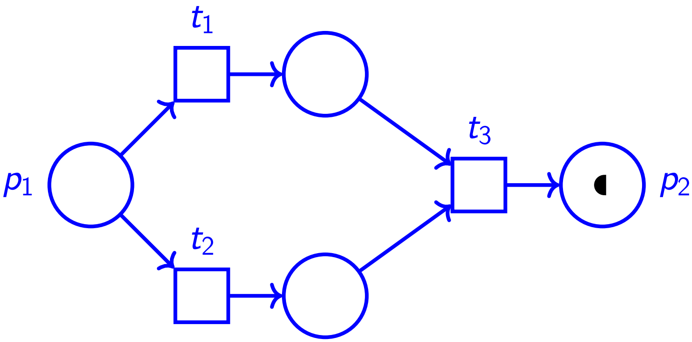

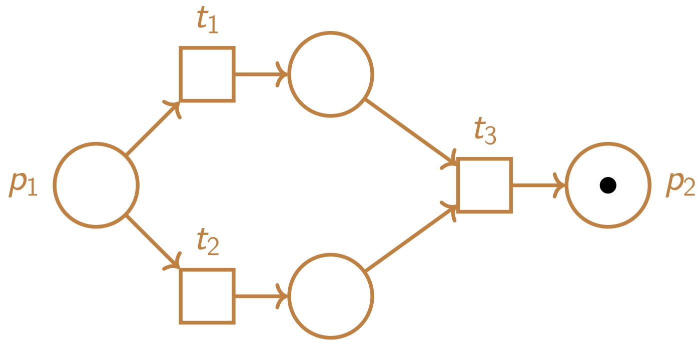

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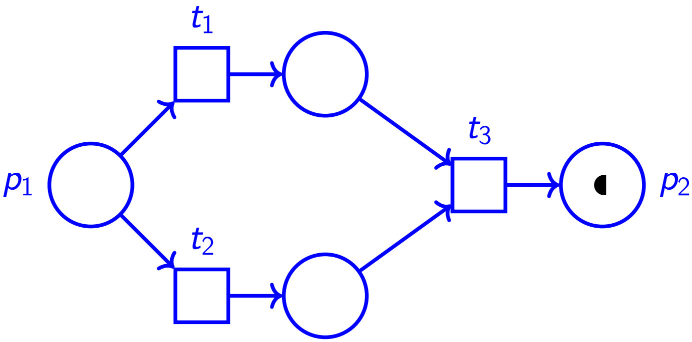

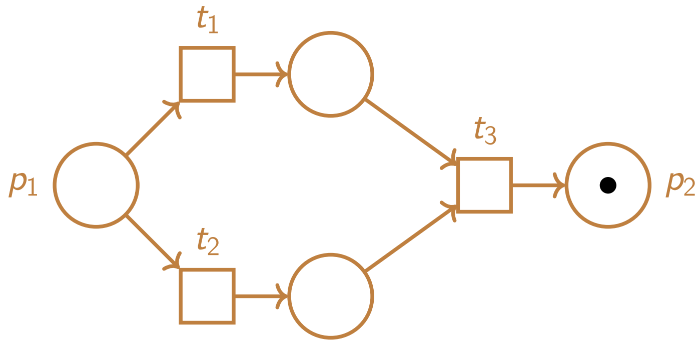

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Continuous reachability $\Leftrightarrow$ Reachability with many tokens

$$\{\mathcal{I} : 1\} \rightarrow_{\mathbb{Q}} m \not\rightarrow_{\mathbb{Q}} \{\mathcal{F} : 1\} \qquad \exists\, k: \{\mathcal{I} : k\} \rightarrow m \not\rightarrow \{\mathcal{F} : k\}$$

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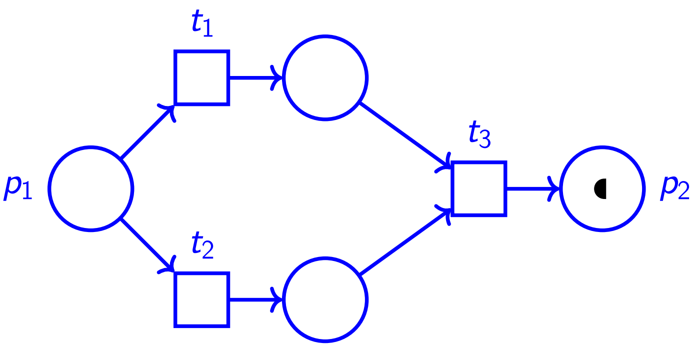

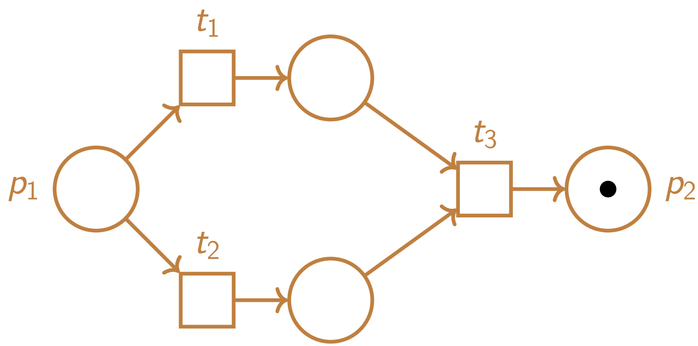

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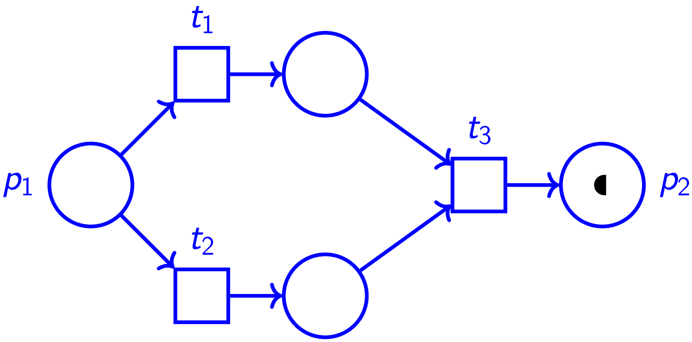

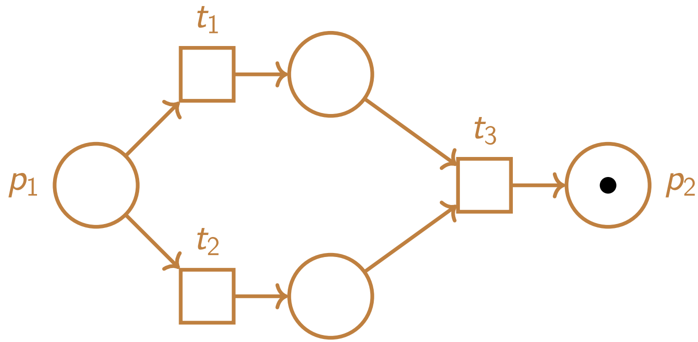

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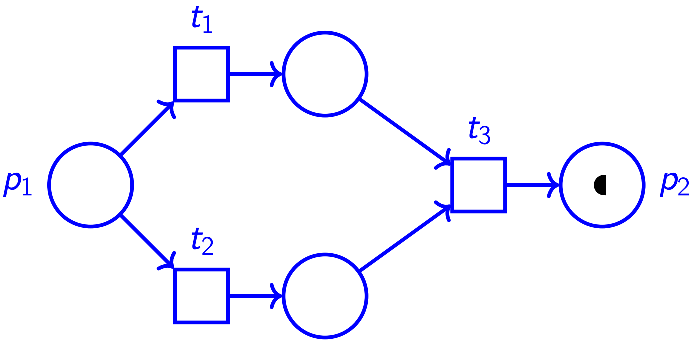

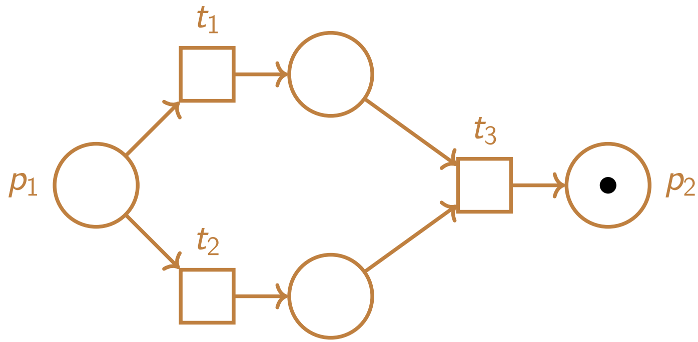

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Complexity of Continuous Soundness

Continuous Reachability: in **PTIME** [Fraca&Haddad, 2013]

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Continuous Soundness: **coNP-complete** [CAV'22]

$\mathbb{Q}\text{-Reach}(N, \{\mathcal{I}\colon 1\}) \subseteq \mathbb{Q}\text{-Reach}(N^{\text{Reversed}}, \{\mathcal{F}\colon 1\})$

Continuous Soundness is a useful criterion

Benchmarks: 1976 industrial nets

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Why is continuous soundness so accurate in practice?

Many instances are actually easy:
Free Choice Workflow Nets

Checking soundness - complexity?

	known results	our work	
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Workflow nets with a restriction on transitions:

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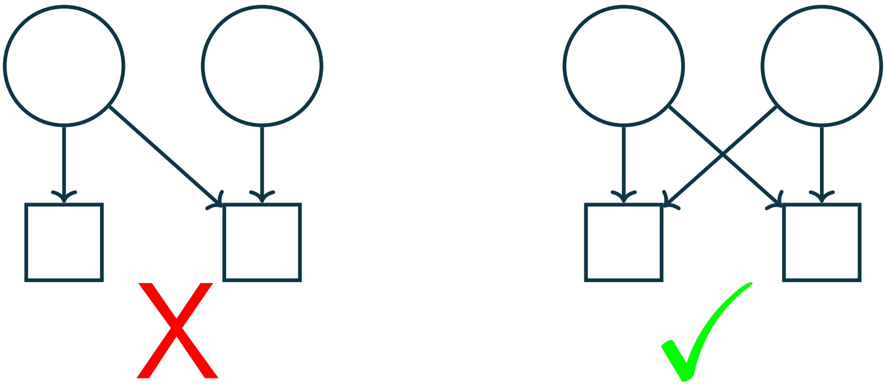

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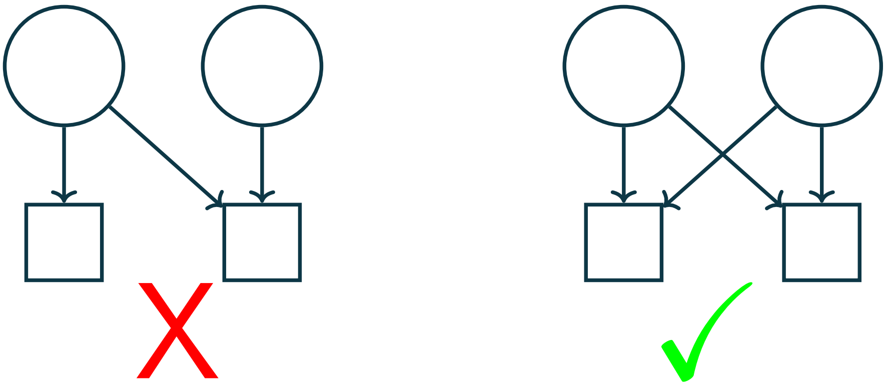

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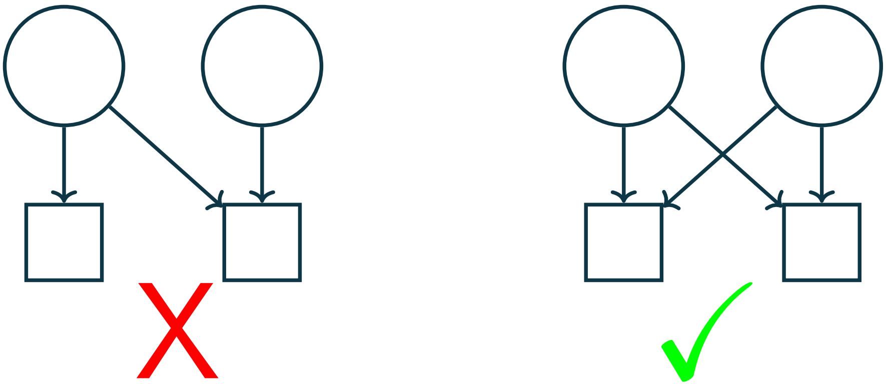

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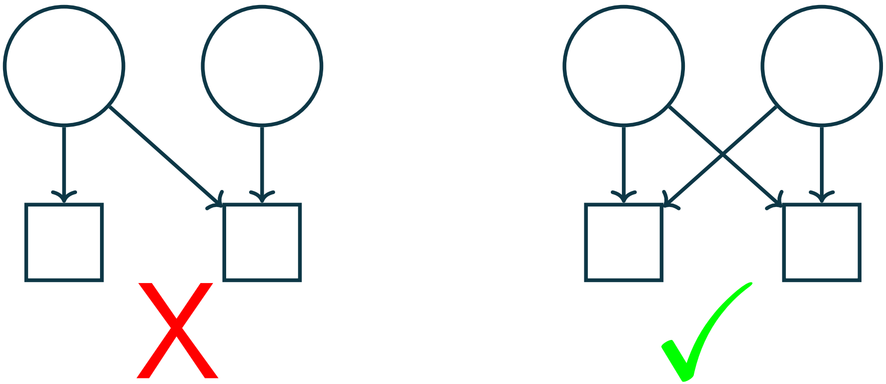

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Continuous soundness is **exact** on free-choice nets

Continuous Soundness on Free Choice nets

Deciding soundness via:

Continuous Soundness vs State Space Exploration

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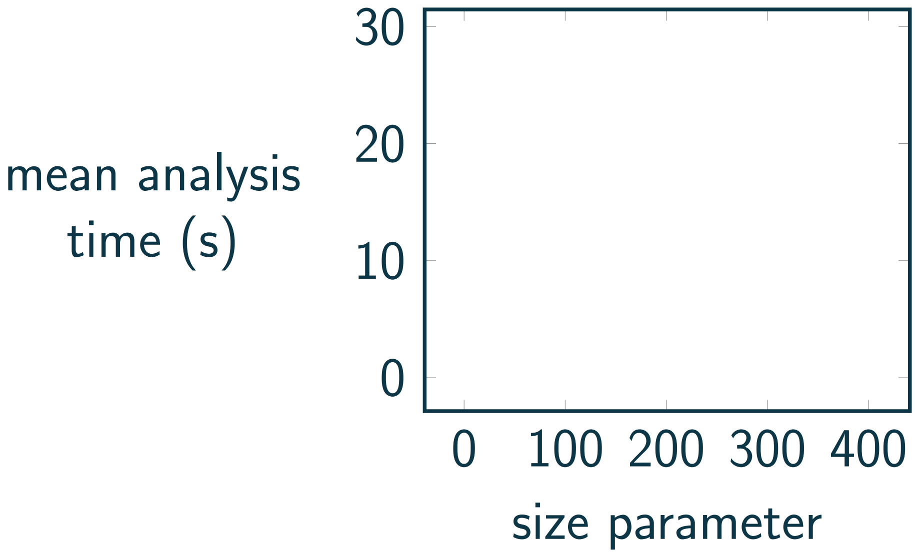

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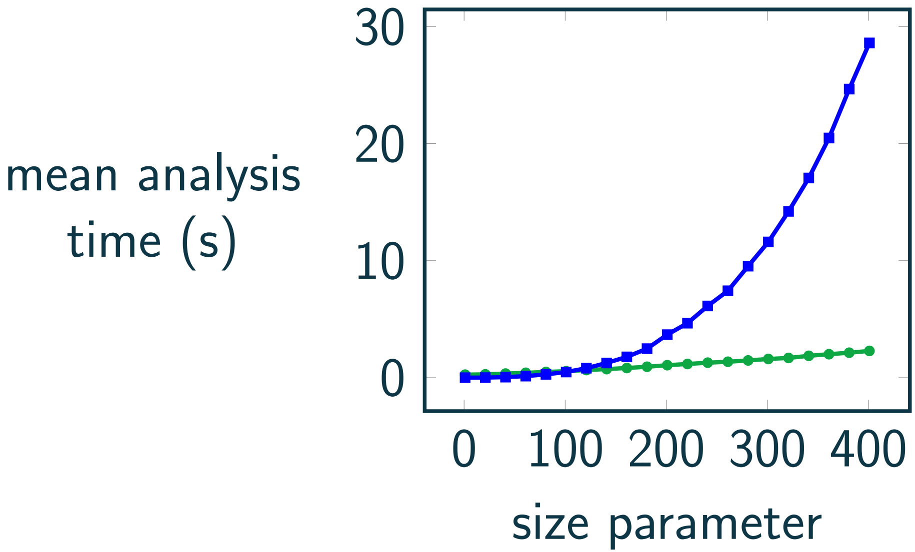

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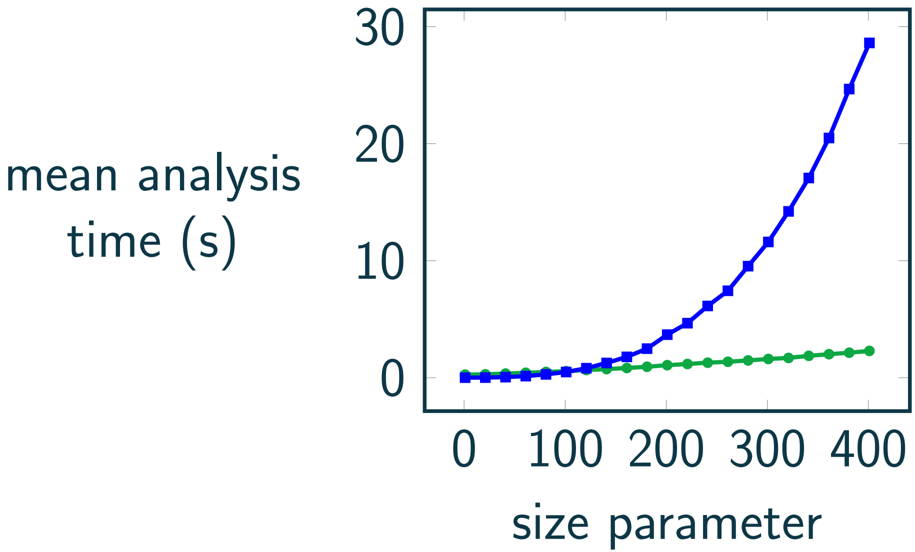

Promising addition to existing techniques for **Free Choice nets**

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Conclusion

Workflow nets formally model **processes**

Soundness is a widely used correctness condition

Variants: **Generalised Soundness**, **Structural Soundness**

Established exact complexities of soundness variants

Continuous soundness: necessary for gen. soundness
and equivalent to soundness variants on **Free-Choice nets**