# The complexity of soundness in workflow nets

## **Philip Offtermatt**

#### Joint work with Michael Blondin and Filip Mazowiecki







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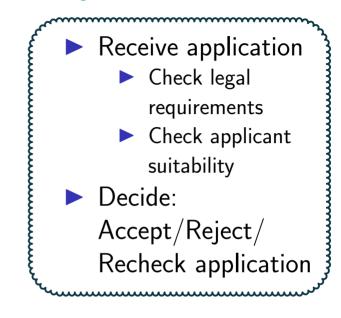
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## **Processes are everywhere!**

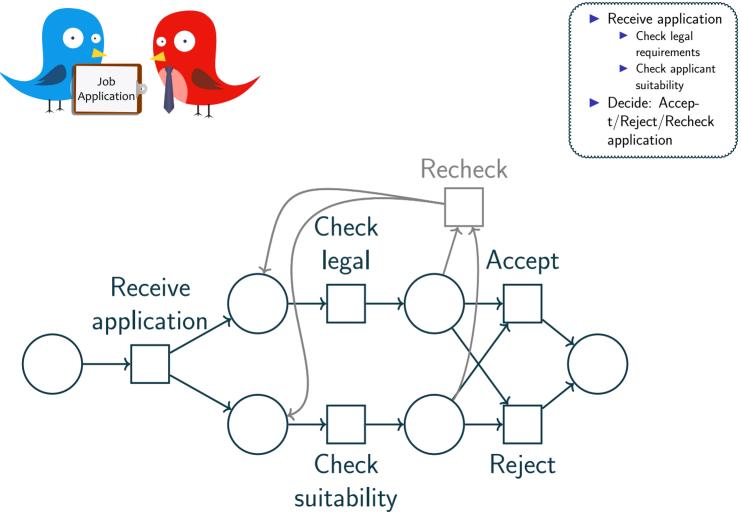


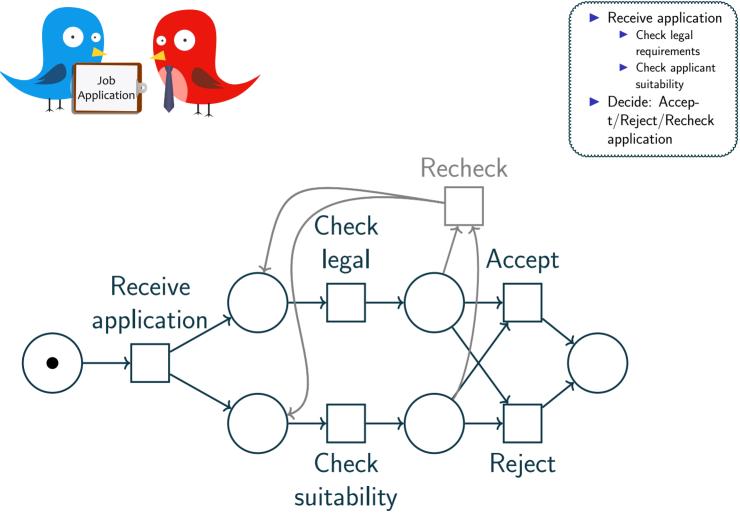
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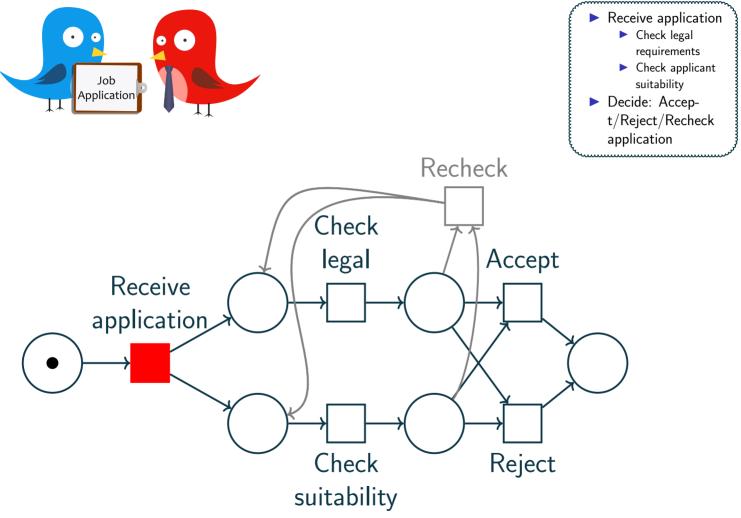


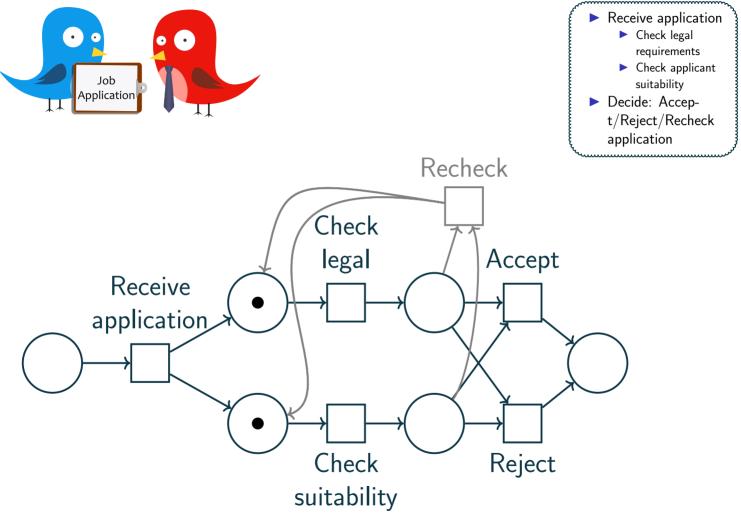


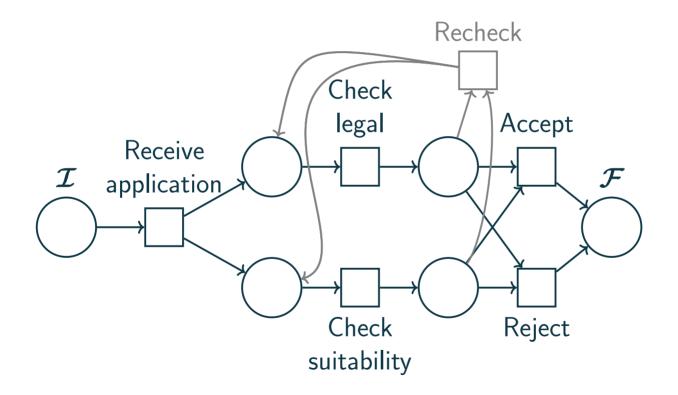
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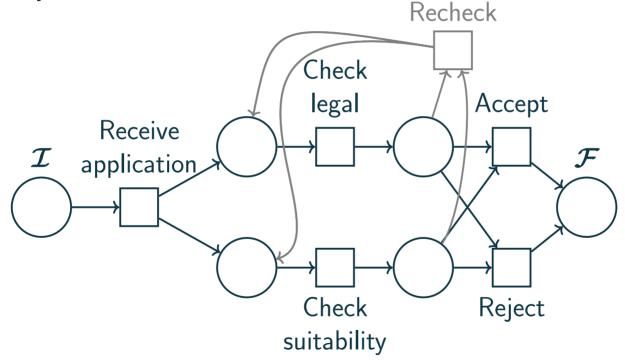






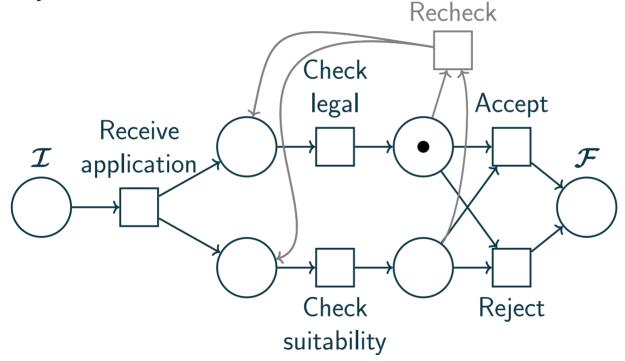
**Option to complete:** 

We should be able to reach a a marking that has tokens only in  ${\cal F}$ 



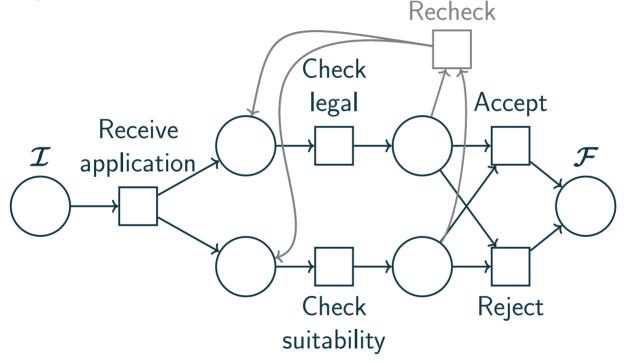
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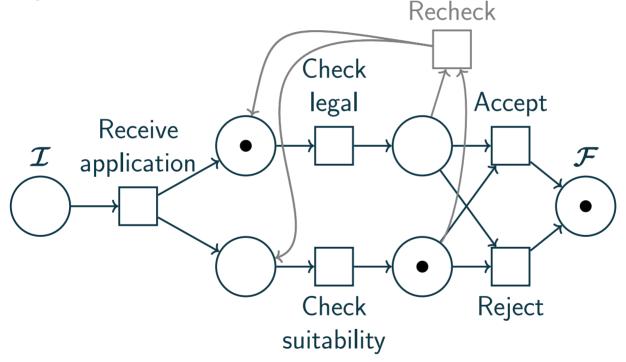
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**Proper completion:** When  $\mathcal{F}$  is marked the rest of the net is empty



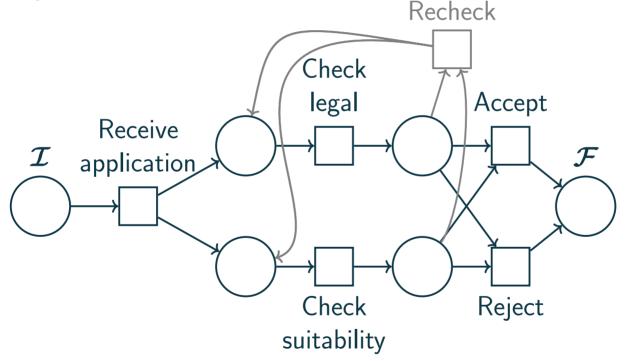
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#### Can we condense these into a single condition?

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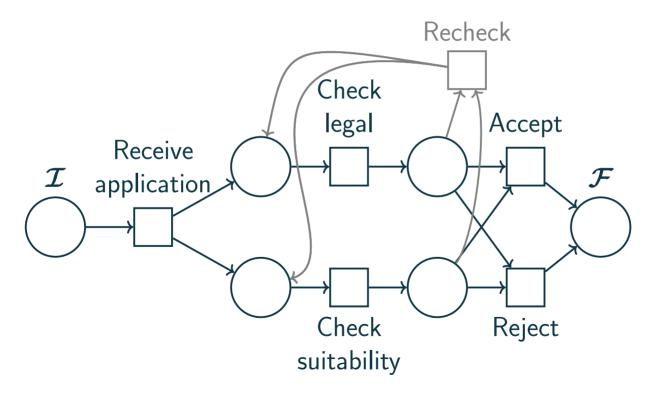
## A concise correctness condition

#### Soundness:

From any marking reachable from  $\{\mathcal{I}: 1\}$ , the final marking  $\{\mathcal{F}: 1\}$  can be reached

 $\forall \mathsf{ runs } \pi \exists \mathsf{ run } \pi' : \{ \mathcal{I} \colon 1 \} \xrightarrow{\pi \pi'} \{ \mathcal{F} \colon 1 \}$ 





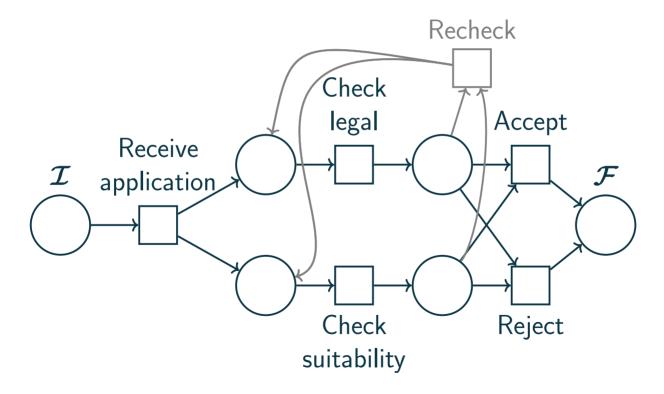
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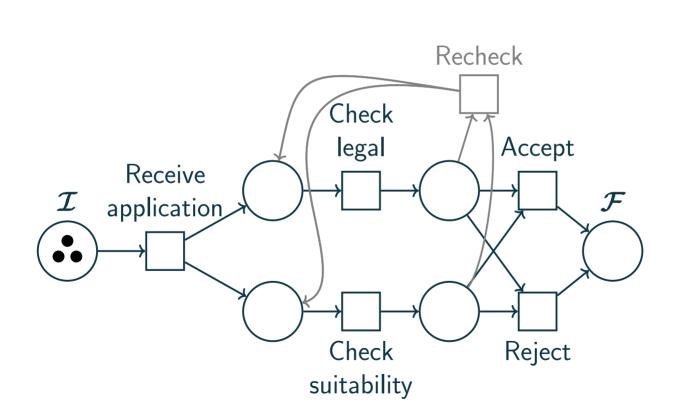




## **Extending soundness**

## *k*-soundness:

From any marking reachable from  $\{\mathcal{I}: \mathbf{k}\}$ , the final marking  $\{\mathcal{F}: \mathbf{k}\}$  can be reached



 $\bullet$ 

Job Application •••

•••

#### *k*-soundness:

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**Generalised soundness:** ∀*k*: *k*-sound

#### *k*-soundness:

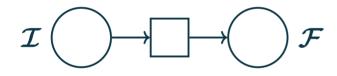
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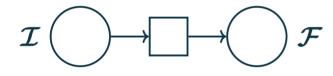
**Generalised soundness:**  $\forall k: k$ -sound **Structural soundness:** ∃*k*: *k*-sound



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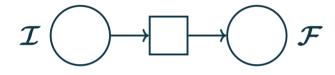


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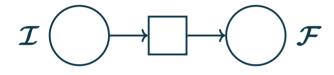


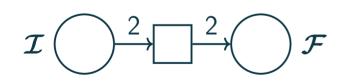


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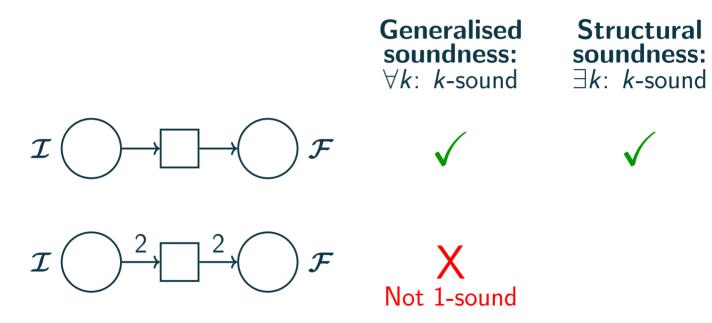
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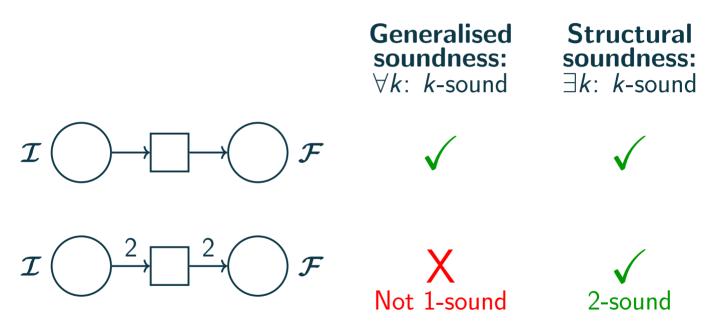




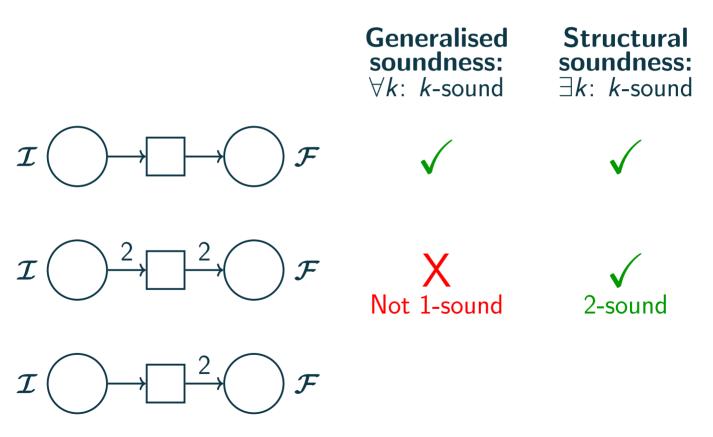
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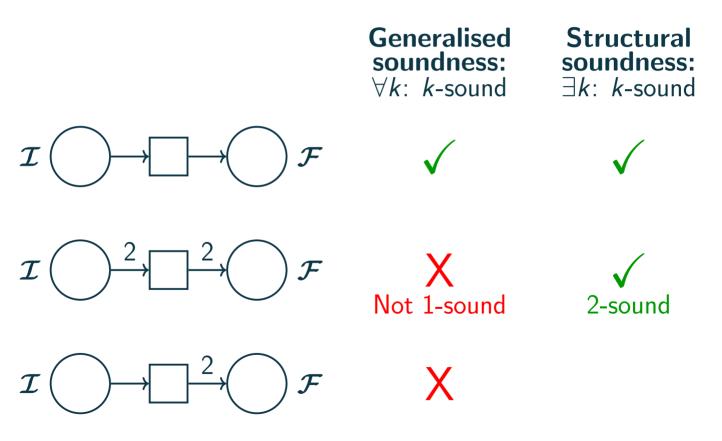
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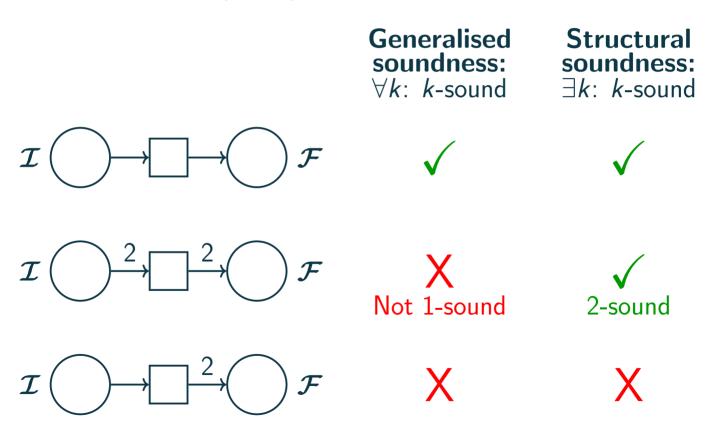
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	known results	our work
k-soundness		
Generalised soundness		
Structural soundness		

	known results	our work
	Decidable	
<i>k</i> -soundness	EXPSPACE-hard?	
	[van der Aalst;'96, '97]	
Generalised		
soundness		
Structural		
soundness		

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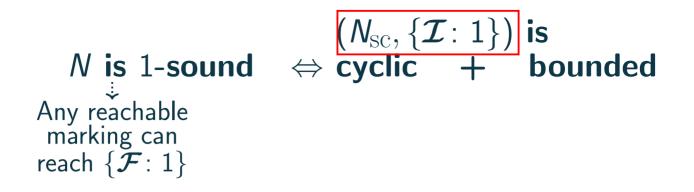
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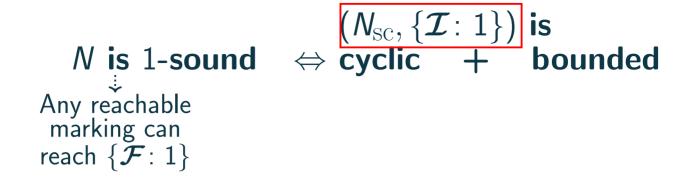
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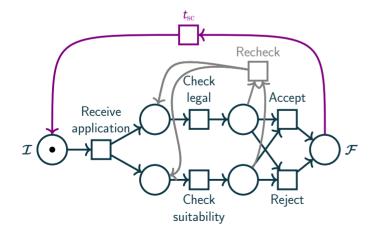
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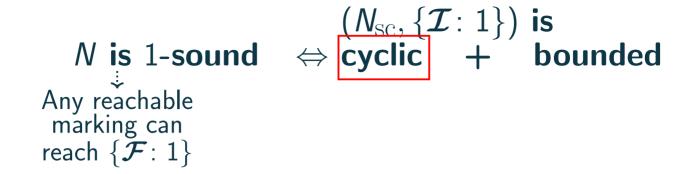
### $\begin{array}{c} (N_{\rm SC}, \{\mathcal{I}:1\}) \text{ is} \\ N \text{ is 1-sound } \Leftrightarrow \text{ cyclic } + \text{ bounded} \end{array}$

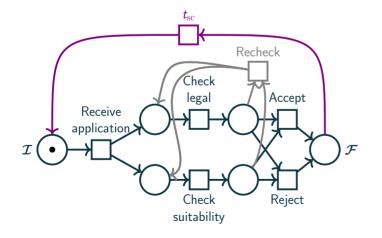
## $(N_{SC}, \{\mathcal{I}: 1\}) \text{ is}$ $N \text{ is } 1\text{-sound} \Leftrightarrow \text{cyclic} + \text{bounded}$ Any reachable marking can $\text{reach } \{\mathcal{F}: 1\}$

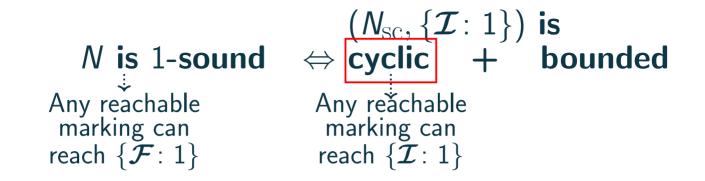


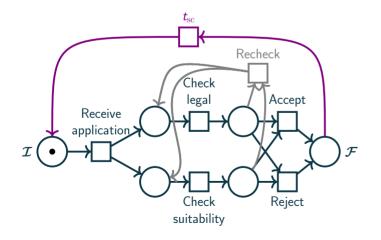




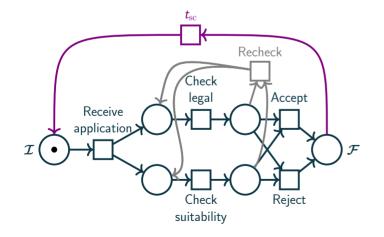


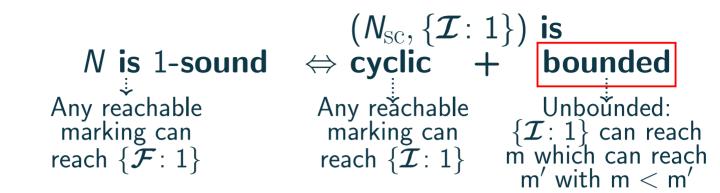


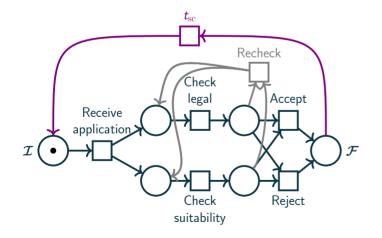


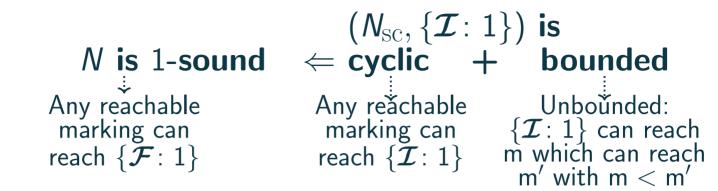


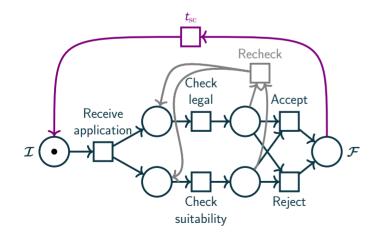


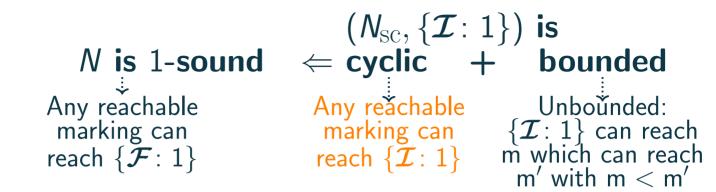




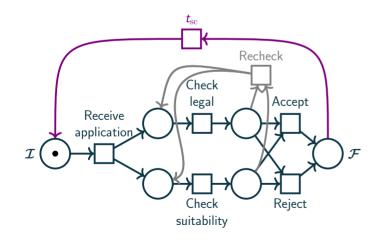




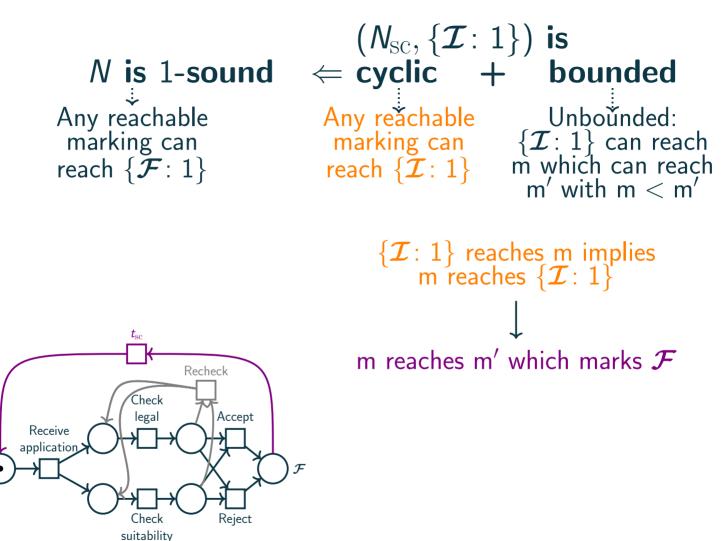


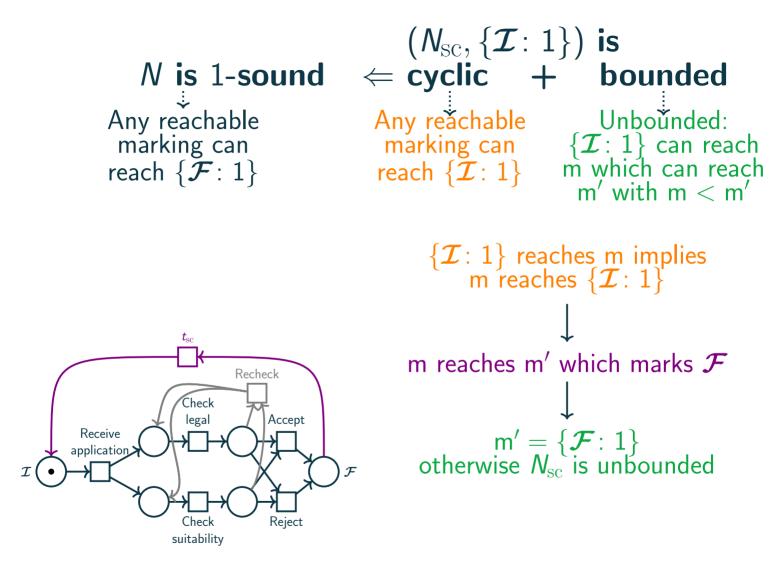


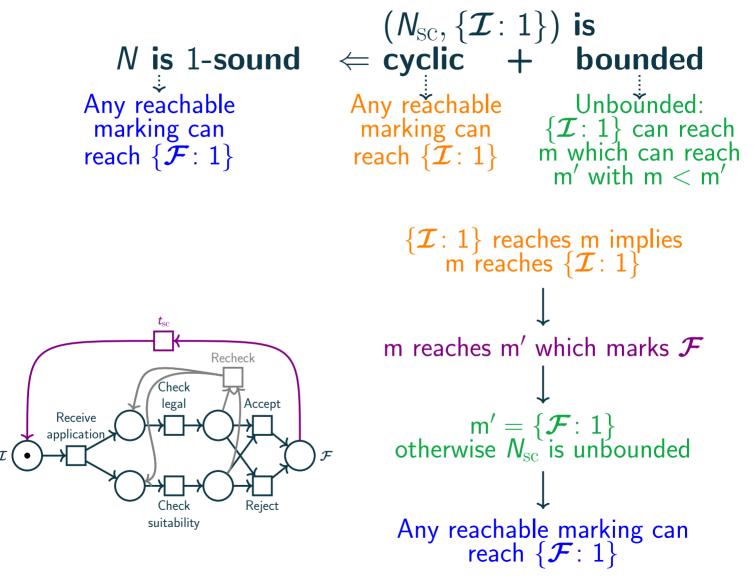
 $\{ \mathcal{I} : 1 \}$  reaches m implies m reaches  $\{ \mathcal{I} : 1 \}$ 



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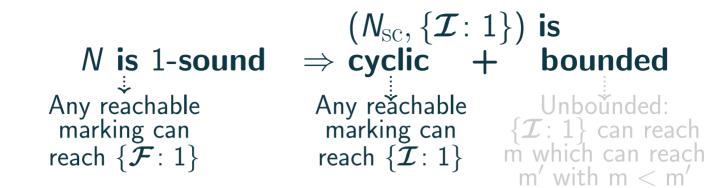


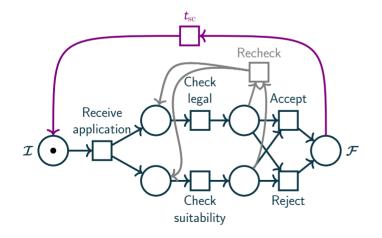


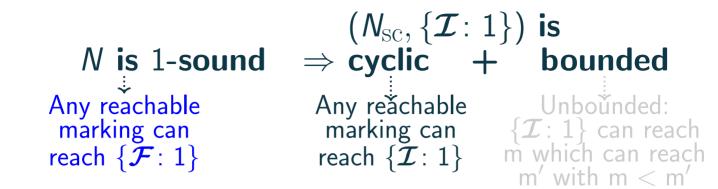


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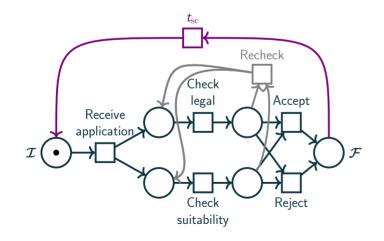
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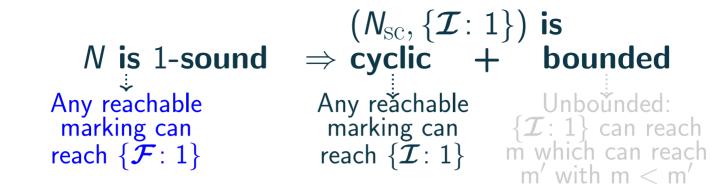


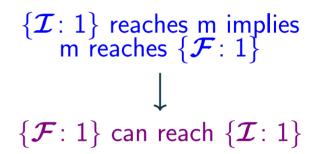


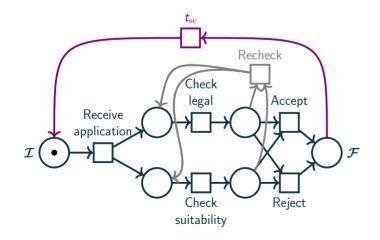


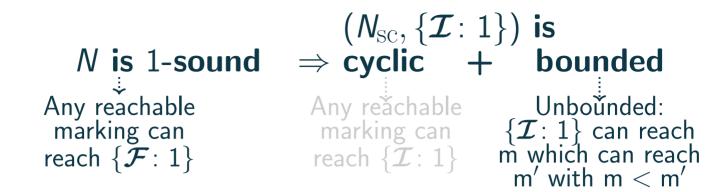
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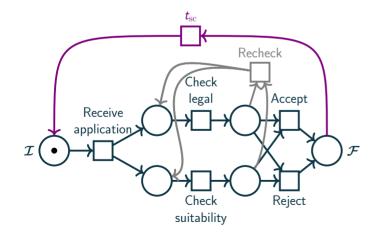






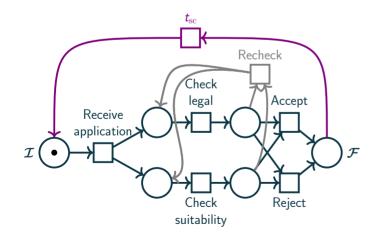


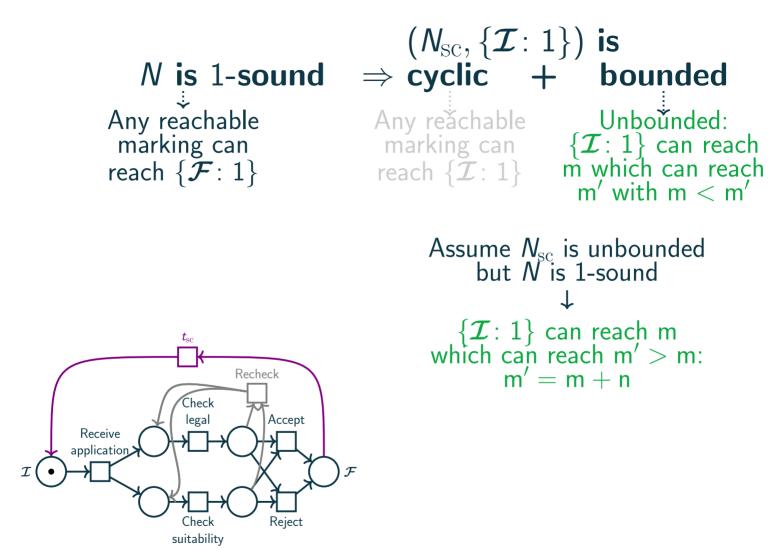


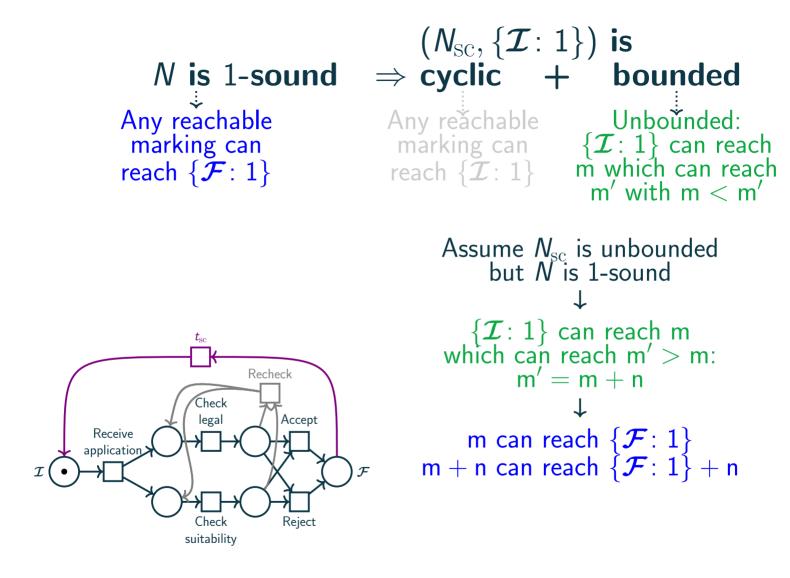


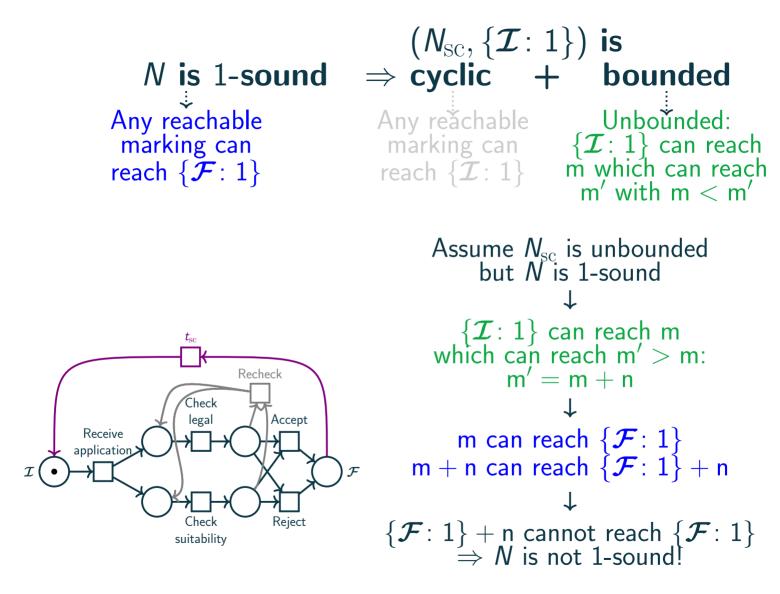
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Assume  $N_{\rm sc}$  is unbounded but N is 1-sound









### $\begin{array}{c} (N_{\rm SC}, \{\mathcal{I}:1\}) \text{ is} \\ N \text{ is 1-sound } \Leftrightarrow \text{ cyclic } + \text{ bounded} \end{array}$

## $(N_{SC}, \{\mathcal{I}: 1\}) \text{ is}$ $N \text{ is 1-sound} \Leftrightarrow \text{cyclic} + \text{bounded}$ $In \text{ EXPSPACE}_{[Bouziane \& Finkel, '97]}$

 $(N_{
m sc},\{\mathcal{I}\colon 1\})$  is *N* is 1-sound  $\Leftrightarrow$  cyclic + bounded In EXPSPACE In EXPSPACE [Bouziane & [Rackoff, '78] Finkel, '97]

$$(N_{SC}, \{\mathcal{I}: 1\}) \text{ is}$$

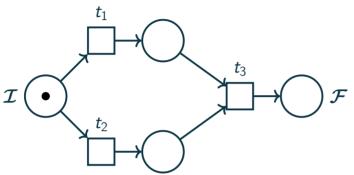
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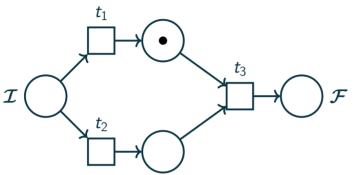
$$\text{In EXPSPACE!} \qquad \text{In EXPSPACE} \qquad \text{In EXPSPACE} \qquad \text{In EXPSPACE} \qquad \text{In EXPSPACE} \qquad \text{[Bouziane \& Finkel, '97]} \qquad \text{In EXPSPACE} \qquad \text{[Rackoff, '78]}$$

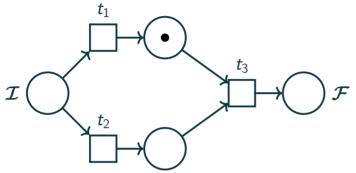
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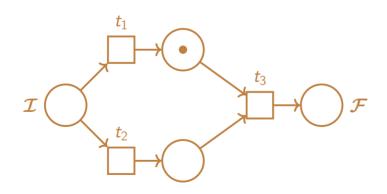
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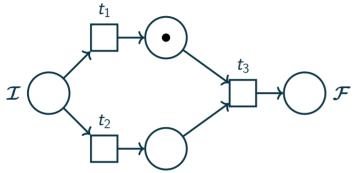
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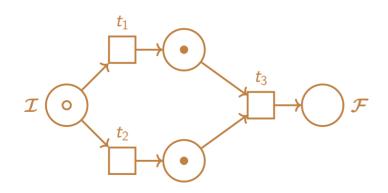




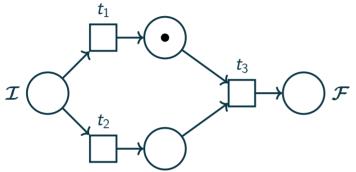


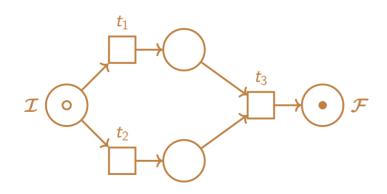




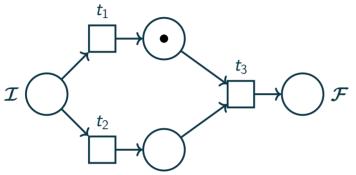


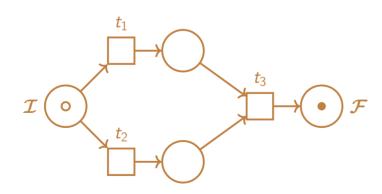
*N* is **generalised sound:**  $\forall k$ : *N* is *k*-sound



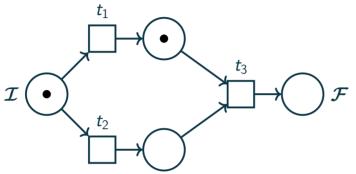


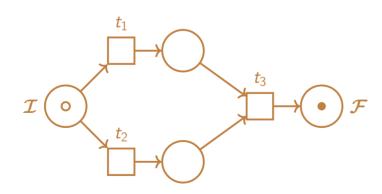
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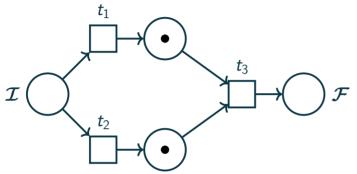


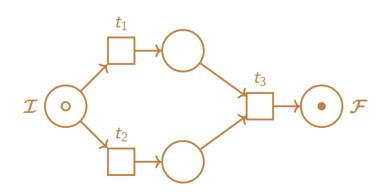
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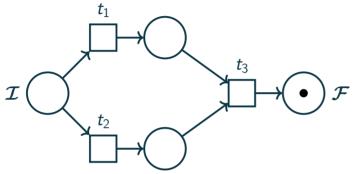


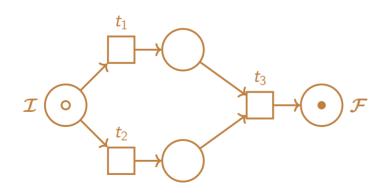
*N* is **generalised sound:**  $\forall k$ : *N* is *k*-sound



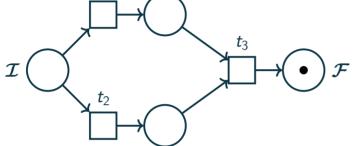


*N* is **generalised sound:**  $\forall k$ : *N* is *k*-sound



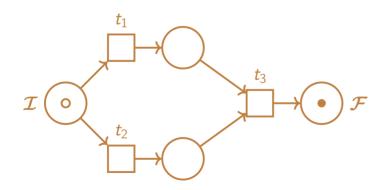


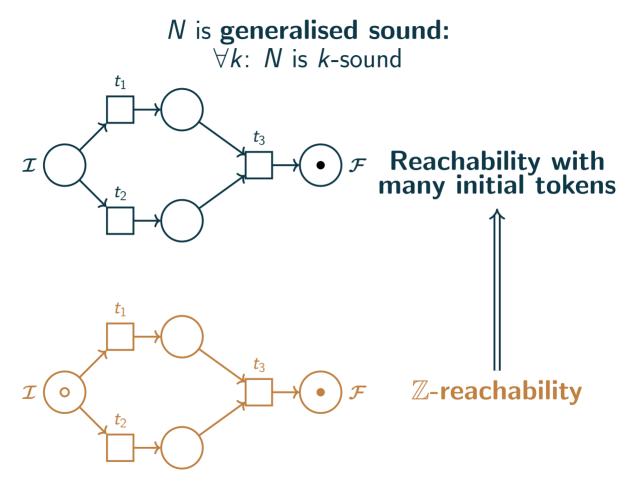
*N* is **generalised sound**:  $\forall k$ : *N* is *k*-sound

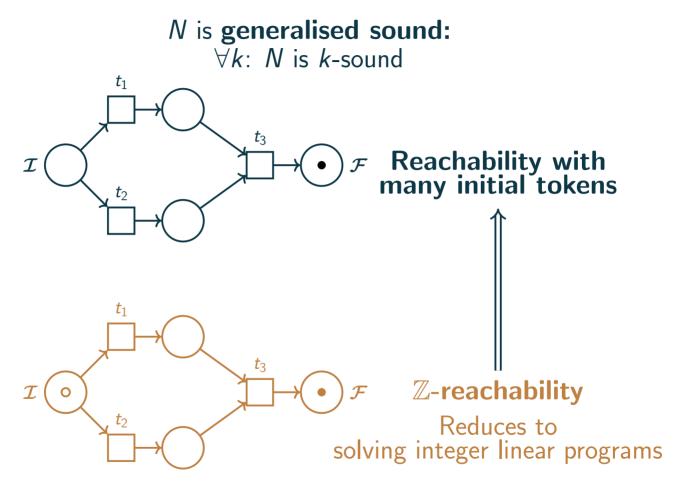


 $t_1$ 

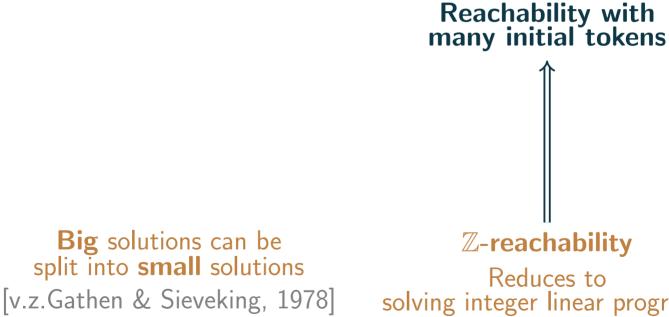
## Reachability with many initial tokens





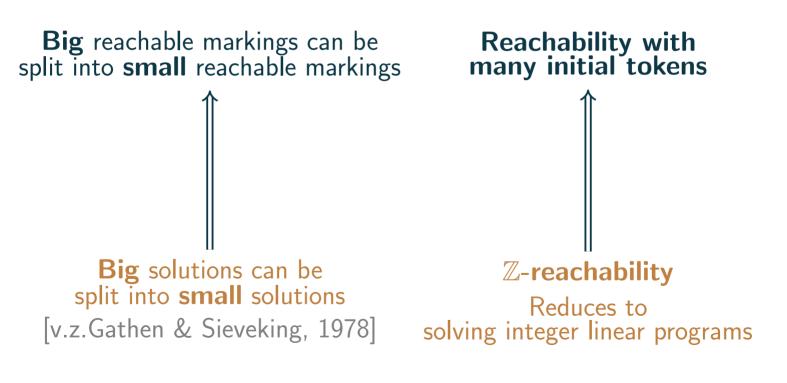


# Reachability with many initial tokens $\mathbb{Z}$ -reachability Reduces to solving integer linear programs



 $\mathbb{Z}$ -reachability Reduces to

solving integer linear programs



Idea: For all Z-reachable small markings check reachability to respective final marking

**Big** reachable markings can be split into **small** reachable markings

**Big** solutions can be split into **small** solutions [v.z.Gathen & Sieveking, 1978]

 $\mathbb{Z}$ -reachability Reduces to solving integer linear programs

**Reachability with** 

many initial tokens

### **Checking soundness - complexity?**

	known results	our work
<i>k</i> -soundness	Decidable EXPSPACE-hard? [van der Aalst;'96, '97]	EXPSPACE- complete
Generalised soundness	Decidable [van Hee et al.;'04]	PSPACE- complete
Structural soundness	Decidable [Țiplea, Marinescu;'04]	EXPSPACE- complete

#### Conclusion

Workflow nets formally model processes

Soundness is an intuitive correctness condition

**Current work**: soundness for non-workflow-nets?